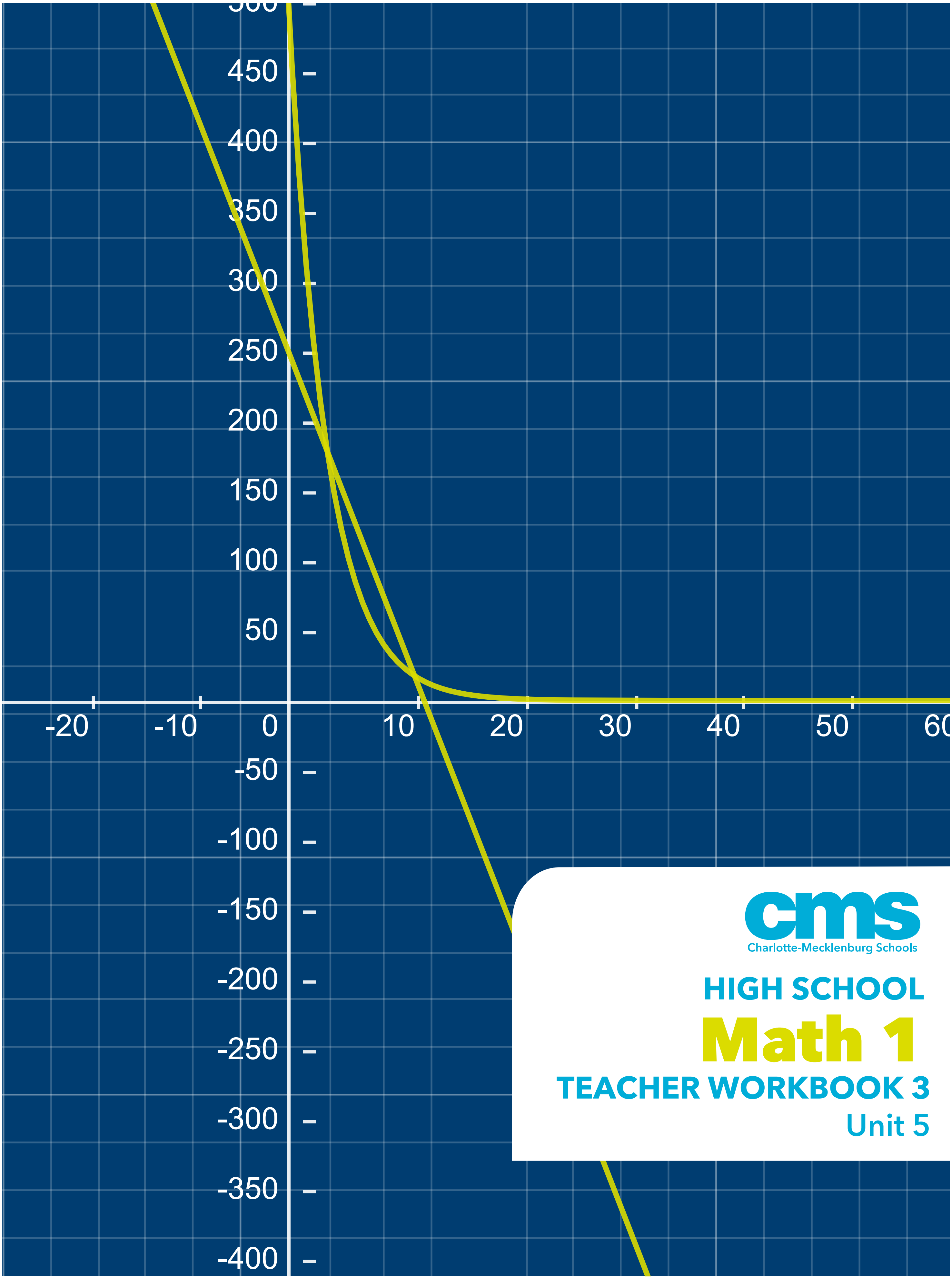


cms
Charlotte-Mecklenburg Schools

HIGH SCHOOL

Math 1

TEACHER WORKBOOK 3

Unit 5

Table of Contents

Math 1

This set of instructional resources aims to provide a math curriculum that students and stakeholders can leverage to promote racial tolerance and oppose racism. Woven throughout the course are experiences for students and stakeholders to examine ideas of social justice, engage in current events, and expand and apply mathematics into everyday life. Reflection, student voice and agency, and high expectations are critical components of this curriculum. As a result, students consistently have opportunities to dig deeper into their worldviews and their identities as mathematicians. It is important to note that some of the topics may encourage passionate conversations and debate among students. Teachers should discuss these opportunities for student discourse during their planning meetings and recognize these possibilities when implementing the curriculum in their own classroom, leveraging their strong classroom cultures and inclusive classroom environments.

Unit 5: Functions

In grade 8, students learned that a function is a rule that assigns exactly one output to each input. They represented functions in different ways—with verbal descriptions, algebraic expressions, graphs, and tables—and used functions to model relationships between quantities, linear relationships in particular.

In this unit, students expand and deepen their understanding of functions. They develop new knowledge and skills for communicating about functions clearly and precisely, investigate different kinds of functions, and hone their ability to interpret functions. Students also use functions to model a wider variety of mathematical and real-world situations.

Lesson 1 provides a refresher on what functions are and what they are not. Students use descriptions, tables, and graphs to reason about the idea of “exactly one output for each input.” In Lesson 2, students learn that function notation is an efficient way to communicate succinctly about functions and devote some focused time to interpret this new notation and use it. They have an opportunity to discuss the amount of garbage produced in the US. In Lessons 3–5, students employ the notation to perform increasingly sophisticated mathematical work: to analyze and compare functions, to write rules of functions (primarily linear functions), to solve for an input, to graph functions, and more.

Next, students focus their attention on graphs of functions and on how they help to tell stories about the relationships between the quantities in the functions. In Lesson 6, students learn to interpret features of graphs and relate them to features of situations, using terms such as “maximum,” “minimum,” and “intercepts” to describe their observations. From a graph, students can see intervals where the values of a function increase or decrease. Students learn to quantify this in Lesson 7, using average rates of change to more precisely describe how quickly these values rise or fall. Students sketch and interpret graphs to depict qualitative behavior of functions beginning in Lesson 8. In Lesson 11, they continue this work, now comparing two functions graphed on the same set of axes.

Lessons 9 and 10 are Checkpoint Lessons. Activities include more practice interpreting and creating graphs of functions revisiting the social phenomenon of food deserts and food insecurity, exploring a micro-modeling task, reviewing simplifying expressions with exponents, and an opportunity to spend time in small group instruction.

In Lessons 12 and 13, students go on to take a closer look at the input and output of a function. They think about possible and reasonable input and output values and learn to identify the domain and range of a function based on contextual and graphical information. In interpreting graphs and in using graphs to find the domain and range of a function, students incorporate prior knowledge from this unit, including maximums/minimums, discrete vs continuous situations, and horizontal/vertical intercepts.

Lesson 14 occurs after administering the Unit 5 assessment and includes post-assessment activities. One of the activities challenges students' biases around what a mathematician looks like.

Instructional Routines

Aspects of Mathematical Modeling: Lessons 8, 9 & 10

Co-Craft Questions (MLR5): Lessons 1, 6, 11

Collect and Display (MLR2): Lessons 2, 4, 11

Compare and Connect (MLR7): Lessons 3, 5, 7, 8

Critique, Correct, Clarify (MLR3): Lessons 2, 12

Discussion Supports (MLR8): Lessons 1, 2, 3, 4, 5, 6, 8, 11, 12, 13

Graph It: Lesson 5

Notice and Wonder: Lessons 4, 7, 9 & 10, 12

Poll the Class: Lessons 6, 7

Round Robin: Lesson 13

Stronger and Clearer Each Time (MLR1): Lessons 7, 8

Take Turns: Lessons 1, 2, 3, 6, 8

Three Reads (MLR6): Lesson 1

Which One Doesn't Belong?: Lessons 8, 13

Lesson 1: Describing and Graphing Situations

PREPARATION

Lesson Goals	Learning Targets
• Interpret descriptions and graphs of functions in context. • Understand that a relationship between two quantities is a function if there is only one possible output for each input. • Use words and graphs to represent relationships that are functions, including identifying the independent and dependent variables.	• I can make sense of descriptions and graphs of functions and explain what they tell us about situations. • I can explain when a relationship between two quantities is a function. • I can identify independent and dependent variables in a function and use words and graphs to represent the function.

Lesson Narrative

In this opening lesson, students recall the meaning of **functions** (which was introduced in grade 8) and encounter examples of functions and their representations in context.

Students consider why certain relationships could be seen as functions while others could not be. By analyzing tables and graphs that represent both functions and non-functions, and by interpreting descriptions of each situation, students are reminded that a function assigns exactly one output value to each input. If an input has more than one possible output, then the relationship cannot be a function.

As they analyze and sketch graphs of functions, students are also reminded that each point on a graph represents an input-output pair of the function. The input is the **independent variable**, and the output is the **dependent variable**. They interpret coordinates on a graph of a function in terms of the quantities in the situation represented.

Students practice making sense of problems and persevering in solving them (MP1) as they look for and explain correspondences between verbal descriptions, tables, and graphs. They engage in aspects of modeling (MP4) as they identify input and output variables in real-life situations and create representations of their relationships.

The work here prepares students to describe and talk about functions more formally in upcoming lessons.

How is the approach of this lesson similar and different from other ways you have taught these concepts or procedures?

Focus and Coherence

Building On	Addressing
NC.6.EE.9: Represent and analyze quantitative relationships by: ● Using variables to represent two quantities in a real-world or mathematical context that change in relationship to one another. ● Analyze the relationship between quantities in different representations (context, equations, tables, and graphs). **NC.8.F.1:** Understand that a function is a rule that assigns to each input exactly one output. ● Recognize functions when graphed as the set of ordered pairs consisting of an input and exactly one corresponding output. ● Recognize functions given a table of values or a set of ordered pairs. **NC.8.F.5:** Qualitatively analyze the functional relationship between two quantities. ● Analyze a graph determining where the function is increasing or decreasing; linear or non-linear. ● Sketch a graph that exhibits the qualitative features of a real-world function.	**NC.M1.F-IF.1:** Build an understanding that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range by recognizing that: ● if f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. ● the graph of f is the graph of the equation $y = f(x)$. **NC.M1.F-IF.4:** Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; and maximums and minimums.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*15 minutes*)
- **Activity 1** (*15 minutes*)
- **Activity 2** (*Optional,10 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U5.L1 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Building On: NC.6.EE.9

The purpose of this bridge is to provide a review of analyzing the relationship between two quantities and identifying the independent and dependent variable. Attention to precision (MP6) is necessary in completing the table in terms of boxes sold.

Student Task Statement

Mai is helping her band collect money to fund a field trip. The band decided to sell boxes of chocolate bars. Each bar sells for $1.50, and each box contains 20 bars. Below is a partial table of monies collected for different numbers of boxes sold.[1]

Boxes sold b	Monies collected m
1	$30.00
2	
3	
4	$120.00
5	
8	
20	
	$1530.00

a. Complete the table above.

b. Write an equation for the amount of money, m, that will be collected if b boxes of chocolate bars are sold.

c. Which is the independent variable and which is the dependent variable?

[1] Adapted from https://tasks.illustrativemathematics.org/

DO THE MATH	PLANNING NOTES

Warm-up: Bagel Shop (*15 minutes*)

Instructional Routine: Co-Craft Questions (MLR5)	
Building On: NC.8.F.1	**Addressing:** NC.M1.F-IF.1

The goal of this warm-up is to activate, through a familiar context, what students know about functions from middle school.

Students first encounter a relationship in which two quantities—the number of bagels bought and price—do not form a function. They see that for some numbers of bagels bought, there are multiple possible prices. The number of bagels bought and the *best* price, however, do form a function, because there is only one possible best price for each number of bagels.

Students contrast the two relationships by:

- making sense of the situation by reasoning about possible prices,
- completing a table of values, and
- analyzing two graphs of the relationships.

Together, these concepts lead to an understanding of the definition of a function.

Step 1

- Have students arrange themselves in pairs or use visibly random grouping.
- With student workbooks closed, use the *Co-Craft Questions* routine by displaying the graphic with bagel prices together with the first line of the Task Statement (A customer at a bagel shop is buying 13 bagels. The shopkeeper says, “That will be $16.25.”) and asking students, “What mathematical questions could be asked about this situation?” Give students a minute to develop one or two questions with their partner. Circulate and identify pairs of students to share their questions with the class. Scribe the questions for all to see, and invite students to answer any that are intriguing.
- Reveal the rest of the Task Statement and give students a few minutes of quiet think time and then time to discuss their thinking with their partner.

CO-CRAFT QUESTIONS	***What Is This Routine?*** Students are presented with a picture, video, diagram, data display, or description of a situation, and their job is to generate one or more mathematical questions that could be asked about the situation. Students then share and compare their questions, as the teacher calls attention to questions that align with the content goals of the lesson. Finally, the “official” question or problem is revealed for students to work on.

Why This Routine? *Co-Craft Questions* (MLR5) allows students to get inside of a context before feeling pressure to produce answers, and creates space for students to produce the language of mathematical questions themselves. Use this routine to spark curiosity about a new mathematical idea or representation, and to elicit everyday student language to brainstorm about the quantitative relationships that might be investigated. During this routine, students use conversation skills and develop meta-awareness of the language used in mathematical questions and problems.

Student Task Statement

A customer at a bagel shop is buying 13 bagels. The shopkeeper says, "That will be $16.25."

Jada, Priya, and Han, who are in the shop, all think it is a mistake.

- Jada says to her friends, "Shouldn't the total be $13.25?"
- Priya says, "I think it should be $13.00."
- Han says, "No, I think it should be $11.25."

Explain how the shopkeeper, Jada, Priya, and Han could all be right.

FRESH BAGELS!

1 bagel	$ 1.25
6 bagels	$ 6.00
9 bagels	$ 8.00
12 bagels	$ 10.00

Step 2

- Select students to share how each person (the shopkeeper, Jada, Priya, and Han) could be correct in calculating the price of 13 bagels.
- As students are sharing, record and display students' reasoning for all to see.
- After the reasoning behind each price is shared, direct students to the table provided. Provide students time to complete the "best price" with their partner.

Student Task Statement

Work with a partner to complete the table:

Number of bagels	Best price
1	
2	
3	
4	
5	
6	
7	

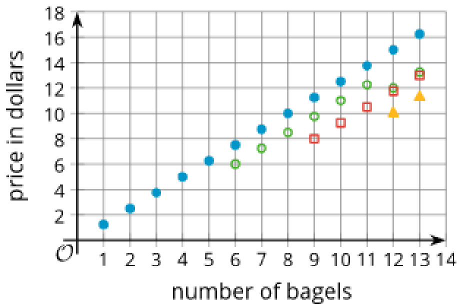
Number of bagels	Best price
8	
9	
10	
11	
12	
13	

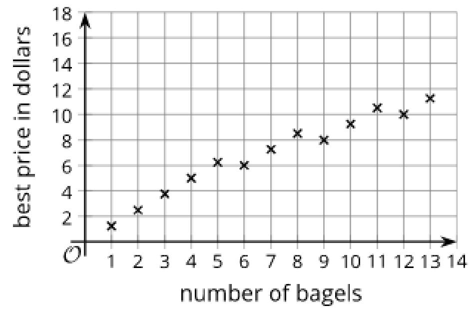
Step 3

- Create a public record of the table of the number of bagels and best price by having students share their answers while you record. This will be used in Lesson 2.
- Consider also displaying a table to summarize the different possibilities for calculating the price of 13 bagels (or the prices for 6 or more bagels).

Number of bagels	Shopkeeper's price	Jada's price	Priya's price	Han's price
1	1.25			
2	2.50			
3	3.75			
4	5.00			
5	6.25			
6	7.50	6.00		
7	8.75	7.25		
8	10.00	8.50		
9	11.25	9.75	8.00	
10	12.50	11.00	9.25	
11	13.75	12.25	10.50	
12	15.00	12.00	11.75	10.00
13	16.25	13.25	13.00	11.25

- Ask students:
  - "'Number of bagels' and 'price' do not form a function, but 'number of bagels' and 'best price' do form a function. Why is this? What do you recall about functions?"
  - "If we graph the relationship between 'number of bagels' and 'price,' what do you think the graph would look like?"
  - "If we graph the relationship between 'number of bagels' and 'best price,' what would the graph look like?"
- After students make some predictions, display the two graphs for all to see. In the first graph, solid blue dots represent the shopkeeper's price; open green circles, Jada's price; red squares, Priya's price; and yellow triangles, Han's price. In the second graph, each X represents the best price for each number of bagels. Using the graphs, ask students:
  - "Why are there no green circles when the number of bagels is fewer than 6?" (Jada's prices start with the cost of 6 bagels.)
  - "Why is price not a function of the number of bagels bought? How do we see this in the graph?" (Because there are multiple possible prices when the number of bages is 6 or greater, price is not a function of the number of bagels bought.)
  - "Why is best price a function of the number of bagels bought? How do we see this in the graph?" (Because there is only one best price for a particular number of bagels, best price is a function of the number of bagels bought.)





- Keep a public record of the table and graph for the number of bagels and best prices for use in Lesson 2.

Advancing Student Thinking: If students need a reinforcement of the term "function," emphasize that a function assigns one output to each input. Clarify that the word "function" in mathematics has a very specific meaning that does not necessarily agree with how "function" is used in everyday life (for instance, as in the sentence: "The function of a bridge is to connect two sides of a river.").

DO THE MATH	PLANNING NOTES

Activity 1: Be Right Back! (*15 minutes*)

Instructional Routine: Three Reads (MLR6)	
Building On: NC.8.F.5	**Addressing:** NC.M1.F-IF.4

This activity further refreshes students' understanding of functions through contextual examples. Here, students are prompted to reason graphically about the relationship between the two quantities in the situation: time in seconds and the distance of a dog from a post. Students also recall the meaning of independent and dependent variables.

Students are given descriptions of the dog's movement while it was attached to a post and asked to sketch corresponding graphs. Along the way, students interpret each point on the graph to mean a particular point in time and a particular distance from the post.

Students later reason that the relationship between time and distance is a function because the dog can only be in one location at any time. For instance, the dog could not be both 5 feet and 1.7 feet away from the post at the same exact time.

RESPONSIVE STRATEGY

Represent the same information through different modalities by using physical objects to represent abstract concepts. Use a three-dimensional prop to represent the situation and the dog's movement. This prop might use a dowel rod for the post and a piece of string or yarn for the leash. Viewing or manipulating the prop can help students to better understand the context and quantities involved.

Supports accessibility for: Conceptual processing; Visual-spatial processing

Step 1

- Ask students to arrange themselves in groups of four or use visibly random grouping.
- Use the *Three Reads* routine to get students started.
  - First Read: Without displaying the problem, read the context aloud to the class:

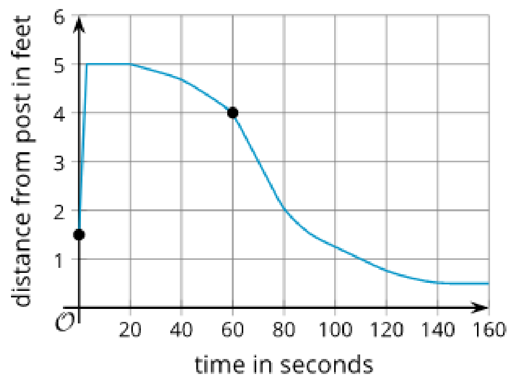
> Three days in a row, a dog owner tied his dog's 5-foot-long leash to a post outside a store while he ran into the store to get a snack. Each time, the owner returned within minutes. The dog's movement each day is described here.
>
> - Day 1: The dog walked around the entire time while waiting for its owner.
> - Day 2: The dog walked around for the first minute and then laid down until its owner returned.
> - Day 3: The dog tried to follow its owner into the store but was stopped by the leash. Then, it started walking around the post in one direction. It kept walking until its leash was completely wound up around the post. The dog stayed there until its owner returned.
> - Each day, the dog was 1.5 feet away from the post when the owner left.
> - Each day, 60 seconds after the owner left, the dog was 4 feet from the post.

    - Ask students: "What is this situation about? What is going on here?" Let students know the focus is just on the situation, not on the numbers. (For example, students might say, "it's about a dog who is tied to a post" or "it's about a dog's movement while the owner ran into a store.")
    - Spend less than a minute scribing their understanding of the situation in a place where all can see. Do not correct students, but do clarify any unfamiliar words; visuals often help (for example, a photo of a dog tied to a post).
  - Second Read: Display the context and ask a student volunteer to read it aloud to the class again.
    - Use the second read to identify quantities and relationships. Ask students what can be counted or measured, without focusing on the values. Listen for, and amplify, the important quantities that vary in relation to each other in this situation: distance from the post and time in seconds.
    - Again, spend less than a minute scribing student responses. Encourage students to identify quantities that are named in the problem explicitly, and any quantities that may be implicit. For each quantity (for example, "1.5 feet"), ask students to add details (for example, "the dog was 1.5 feet away from the post when the owner left"). Again, this is brief.
  - Third Read: Invite students to read the context again to themselves, together with the problem: Sketch a graph that could represent the dog's distance from the post, in feet, as a function of time, in seconds, since the owner left.
    - After the third read, ask students to brainstorm possible strategies to complete this problem. This helps students connect the language in the context with the reasoning needed to solve the problem while maintaining the cognitive demand of the task.
    - Ask: "How might we approach this problem? What is the first thing you will do?"
    - Spend 1–2 minutes scribing student ideas as they brainstorm possible starting points. Be sure to stop any students who begin to share a complete solution; the goal is to crowdsource some starting points.

THREE READS	***What Is This Routine?*** A word problem is read three times, with a different question posed with each read: (1) What is this situation about?; (2) What can be counted or measured in this situation?; (3) How might we approach this problem, or what is the first thing you will do to get started? ***Why This Routine?*** *Three Reads* (MLR6) gives students a chance to use everyday language to help each other make sense of the context—and the language—of a word problem before jumping down a solution path. Use this routine to ensure that students know what they are being asked to do, and to create an opportunity for students to reflect on the general structure of quantitative situations and on the ways mathematical questions are presented. This routine supports reading comprehension of problems and meta-awareness of mathematical language. It also supports negotiating information in a text with peers through mathematical conversation.

Step 2

- Prior to students working in their groups, ask students: “What are some features of a graph that could represent the dog’s movement on Day 3?”
    - Display a possible graph (an example is shown here), and ask students how the graph represents the dog’s movement. Discuss the features in the graph.
    - Encourage students to identify quantities in context and consider which quantities are independent and dependent in the situation. If needed, remind students that a quantity that is an input for a function is called an “independent variable,” and a quantity that is an output is called a “dependent variable.” In this case, time is independent and distance from the post is dependent.



- Tell students that in each group, two students should work on the graph of the dog’s movement on Day 1, and two students should work to graph the dog’s movement on Day 2.

Monitoring Tip: As students work, notice if any students create a graph with a vertical value greater than 5 or a graph that does not go through the two given points. If many graphs show these features, discuss with the class why the former is not possible and why the latter does not match the given descriptions.

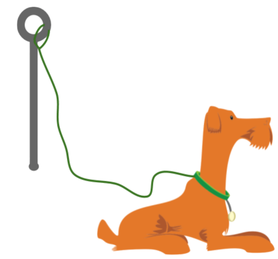
Step 3

- Select some students to share their graphs and a brief explanation of how the graphs match the descriptions. Take opportunities for students to explain what specific points on their graphs represent. If it doesn’t come up naturally, ask students what all of the Day 2 graphs have in common after 60 seconds. (They are all horizontal lines.)
- Ask students: “Why is the relationship between the time since the owner left and the dog’s distance from the post a function?” Find as many explanations as possible as to why this is a function.
    - If not mentioned by students, emphasize that a function is a relationship in which one output is assigned to every input, by highlighting that:
        - The dog can only be in one spot at a time and cannot have more than one distance away from the post at any time.
        - On each graph, we can see that only one value for distance corresponds to each value for time.

- Ask students:
  - "In this situation, what is the input and what is the output?" (It makes sense for time in seconds to be the input and distance from the post to be the output.)
  - "Is it possible for the distance to be the input and the time to be the output? Why or why not?" (It is possible, but if the distance is the input, the relationship is no longer a function. This is because there could be multiple possible times at which the dog is a certain distance away from the post.)

Student Task Statement

Three days in a row, a dog owner tied his dog's 5-foot-long leash to a post outside a store while he ran into the store to get a snack. Each time, the owner returned within minutes.



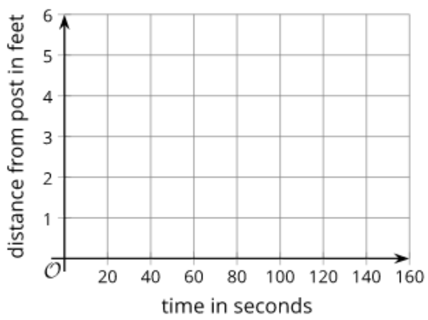
The dog's movement each day is described here.

- Day 1: The dog walked around the entire time while waiting for its owner.
- Day 2: The dog walked around for the first minute and then laid down until its owner returned.
- Day 3: The dog tried to follow its owner into the store but was stopped by the leash. Then, it started walking around the post in one direction. It kept walking until its leash was completely wound up around the post. The dog stayed there until its owner returned.
- Each day, the dog was 1.5 feet away from the post when the owner left.
- Each day, 60 seconds after the owner left, the dog was 4 feet from the post.

Your teacher will assign one of the days for you to analyze.

Sketch a graph that could represent the dog's distance from the post, in feet, as a function of time, in seconds, since the owner left.

Day ________



Are You Ready For More?

From the graph, is it possible to tell how many times the dog changed directions while walking around? Explain your reasoning.

DO THE MATH

PLANNING NOTES

Activity 2: Talk about a Function (*Optional*, *10 minutes*)

Instructional Routines: Discussion Supports (MLR8) - Responsive Strategy; Take Turns	
Building On: NC.8.F.5	**Addressing:** NC.M1.F-IF.1; NC.M1.F-IF.4

This activity gives students a chance to use mathematical language to describe relationships that are functions and to practice sketching a graph of a function given a description. The context is similar to the previous activity, but the quantities are different.

Deciding on which variable is independent and which is dependent, as well as sketching a graph of the relationship, engages students in aspects of modeling (MP4). They require students to make sense of quantities, consider how they are related, and think about what values are reasonable. Using the language of function to articulate the relationship between variables is an opportunity to attend to precision (MP6).

When sketching a graph for the function that defines the number of barks, students are likely to create either a discrete graph or a continuous graph. (Because the total number of barks cannot be fractional, creating a step graph would most accurately represent it as a function of time, but students are not expected to do so at this point.)

Step 1

- Have students arrange themselves in pairs or use visibly random grouping.
- Explain to students their task is to analyze two pairs of quantities from a familiar situation, but working with their partner, they should each choose a different pair of quantities.
- Provide students quiet work time followed by time for partners to *Take Turns* sharing their functions and representations.
- As partners are sharing, the partner who is listening should listen for the following information:
  - How did their partner decide which quantity is independent (the input) and which is dependent (the output)?
  - How did they decide what makes ___ a function of ____?
  - How does the graph represent the relationship between ___ and ___?
  - On the graph, how was the scale for each axis determined?
- Encourage students to notice any part of their own or their partner's statement or graph that may not seem reasonable in the situation, and then think about what might be more reasonable. (For instance, it is not reasonable for a dog to bark 1,000 times in 2 minutes, and it is not reasonable for total distance to decrease over time.)

RESPONSIVE STRATEGY

Use this routine to support partner discussion as students analyze two pairs of quantities. Display the following sentence frames for all to see: "____ is the input or output because . . .", "I noticed ____, so. . .", or "To determine a scale on the graph, I. . . ." Encourage students to challenge each other when they disagree. This will help students use mathematical language to describe relationships that are functions.

Discussion Supports (MLR8)

RESPONSIVE STRATEGY

Invite students to engage in active listening during the partner activity by providing a two-column graphic organizer with the questions in one column and a blank space for recording their partners strategies in the other column.

Supports accessibility for: Language; Organization

DISCUSSION SUPPORTS	***What Is This Routine?*** The teacher uses multi-modal strategies for helping students comprehend and generate language and ideas, such as sentence frames, word walls, images and videos, revoicing, choral response, gesture, and graphic organizers. The strategies can be combined and used together with any of the other routines. ***Why This Routine?*** *Discussion Supports* (MLR8) foster rich and inclusive discussions about mathematical ideas, representations, contexts, and strategies. Use *Discussion Supports* to make classroom communication accessible, to increase meta-awareness of language, and to demonstrate strategies students can use to enhance their own communication and construction of ideas.

TAKE TURNS

What Is This Routine? Students work with a partner or small group. They take turns in the work of the activity, whether it be spotting matches, explaining, justifying, agreeing or disagreeing, or asking clarifying questions. If they disagree, they are expected to support their case and listen to their partner's arguments. The first few times students engage in these activities, the teacher should demonstrate, with a partner, how the discussion is expected to go. Once students are familiar with these structures, less set-up will be necessary. While students are working, the teacher can ask students to restate their question more clearly or paraphrase what their partner said.

Why This Routine? Building in an expectation, through the *Take Turns* routine, that students explain the rationale for their choices and listen to another's rationale deepens the understanding that can be achieved through these activities. Specifying that students take turns deciding, explaining, and listening limits the phenomenon where one student takes over and the other does not participate. Taking turns can also give students more opportunities to construct logical arguments and critique others' reasoning (MP3).

***Advancing Student Thinking*:** Because we often identify the input of a function first, some students may think that the independent variable always goes first in the statement "_____ is a function of _____."

Explain that we can think of the phrase "is a function of" to mean "depends on." Suggest that they use "depends on" to describe the relationship between the quantities both ways and see which one makes more sense. For example, "The dog's distance from the post depends on the time since the owner left," or "the time since the owner left depends on the dog's distance from the post." The former makes more sense, so we can say, "The dog's distance from the post is a function of time since the owner left."

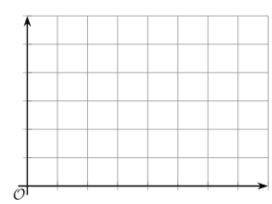
Student Task Statement

Here are two pairs of quantities from a situation you've seen in this lesson. Each pair has a relationship that can be defined as a function.

- Time, in seconds, since the dog owner left and the total number of times the dog has barked.
- Time, in seconds, since the owner left and the total distance, in feet, that the dog has walked while waiting.

Choose one pair of quantities and express their relationship as a function.

1. In that function, which variable is independent? Which one is dependent?
2. Write a sentence of the form "____ is a function of _____."
3. Sketch a possible graph of the relationship on the coordinate plane. Be sure to label and indicate a scale on each axis, and be prepared to explain your reasoning.



Step 2

- For each situation, select one or two students who drew different graphs to display them for all to see. Ask the students to briefly explain how they decided which quantity should be the input and which should be the output, as well as what the graph should look like.

DO THE MATH

PLANNING NOTES

Lesson Debrief (*5 minutes*)

The purpose of this lesson debrief is for students to discuss and refine the necessary conditions needed for a relationship between two quantities to be defined as a function.

Choose whether students should first have an opportunity to reflect in their workbooks or talk through these questions with a partner.

Refer back to the situations in the lesson activities as needed.

- "In the situation that involved bagels, why was the relationship between the number of bagels and the best price a function?" (There is only one possible best price for each number of bagels.)
- "In the situation that involved time and the dog's distance from the post, suppose the distance is the input and the time is the output. Is it possible that an input of 27 feet might have outputs of 50 seconds, 55 seconds, and 60 seconds?" (Yes)
- "Is time a function of the distance of the dog from the post? Why or why not?" (Probably not, because for any distance of the dog from the post, many possible numbers of seconds could have passed.)

PLANNING NOTES

Student Lesson Summary and Glossary

A relationship between two quantities is a **function** if there is exactly one output for each input. We call the input the **independent variable** and the output the **dependent variable**.

Function: A function takes inputs from one set and assigns them to outputs from another set, assigning exactly one output to each input. For example, if apples cost $1.30 per pound, then the relationship between number of apples bought and price paid is a function. An input of 5 apples has an output of $6.50 and no other price.

Independent variable: A variable representing the input of a function. If apples cost $1.30 per pound, then number of apples bought is the independent variable that determines the price paid for those apples.

Dependent variable: A variable representing the output of a function. If apples cost $1.30 per pound, then price paid for the apples is the dependent variable: it depends on the number of apples bought.

Let's look at the relationship between the amount of time since a plane takes off, in seconds, and the plane's height above the ground, in feet.

- These two quantities form a function if time is the independent variable (the input) and height is the dependent variable (the output). This is because at any amount of time since takeoff, the plane could only be at one height above the ground.

  For example, 50 seconds after takeoff, the plane might have a height of 180 feet. At that moment, it cannot be both 180 feet and 95 feet above the ground.

  For any input, there is only one possible output, so the height of the plane is *a function of* the time since takeoff.

- The two quantities do not form a function, however, if we consider height as the input and time as the output. This is because the plane can be at the same height for multiple lengths of times since takeoff.

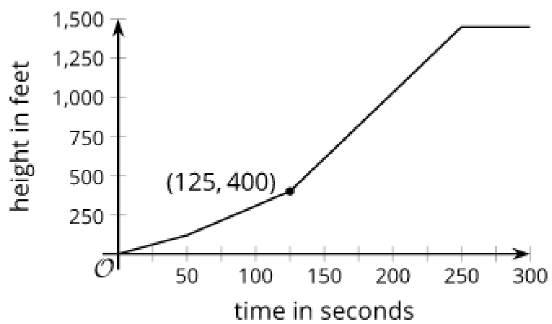
  For instance, it is possible that the plane is 1,500 feet above ground when 300 seconds have passed AND when 425 seconds have passed. In fact, if the plane moves up and down a lot, there could be many other times at which the plane is 1,500 feet above ground.

  For any input, there are multiple possible outputs, so the time since takeoff is *not a function of* the height of the plane.

Functions can be represented in many ways—with a verbal description, a table of values, a graph, an expression or an equation, or a set of ordered pairs.

When a function is represented with a graph, each point on the graph is a specific (input, output) pair..

Here is a graph that shows the height of a plane as a function of time since takeoff.



The point $(125, 400)$ tells us that 125 seconds after takeoff, the height of the plane is 400 feet.

Cool-down: The Backyard Pool (*5 minutes*)

Addressing: NC.M1.F-IF.1; NC.M1.F-IF.4

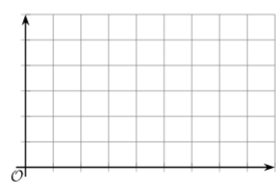
Cool-down Guidance: More Chances
Students will have more opportunities to understand the mathematical ideas in this cool-down, so there is no need to slow down or add additional work to the next lessons. Instead, use the results of this cool-down to provide guidance for what to look for and emphasize over the next several lessons to support students in advancing their current understanding.

Cool-down

A parent is using a garden hose to fill up a small inflatable pool for her young child. The pool has a capacity of 90 gallons. She turns the water off after 5 minutes, but she leaves the hose in the pool for another 3 minutes before putting it away.

In this situation, the relationship between the gallons of water in the pool and the 8 minutes since the parent started filling the pool can be seen as a function.

1. In that function, which variable is independent? Which one is dependent?
2. Write a sentence of the form "____is a function of ____."
3. Sketch a possible graph of the relationship on the coordinate plane. Be sure to label and indicate a scale on each axis.



Student Reflection:

In your own words, explain how you can distinguish between the independent and dependent variable?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

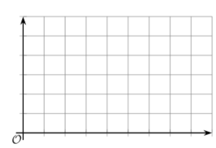
TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

What connections did students make between the different strategies shared? What questions did you ask to help make the connections more visible?

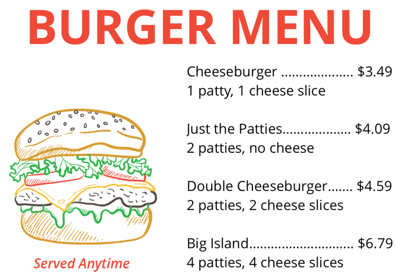
Practice Problems

1. The relationship between the amount of time a car is parked, in hours, and the cost of parking, in dollars, can be described with a function.
    a. Identify the independent variable and the dependent variable in this function.
    b. Describe the function with a sentence of the form "____ is a function of ____."
    c. Suppose it costs $3 per hour to park, with a maximum cost of $12. Sketch a possible graph of the function. Be sure to label the axes.
    d. Identify one point on the graph and explain its meaning in this situation.



2. The prices of different burgers are shown on this sign.

    Based on the information from the menu, is the price of a burger a function of the number of patties? Explain your reasoning.



3. The distance a person walks, d, in kilometers, is a function of time, t, in minutes, since the walk begins.

   Select **all** true statements about the input variable of this function.

   a. Distance is the input.
   b. Time of day is the input.
   c. Time since the person starts walking is the input.
   d. t represents the input.
   e. d represents the input.
   f. The input is not measured in any particular unit.
   g. The input is measured in hours.

4. It costs \$3 per hour to park in a parking lot, with a maximum cost of \$12.

   Explain why the amount of time a car is parked is not a function of the parking cost.

5. The Panthers, Charlotte's professional football team, track their score as a function of the amount of time passed in a football game. The table below gives the output scores, y, for a given time, x:

Time passed (in minutes)	0	10	20	30	40	50	60
Points scored	0	10	13	16	23	36	36

   If the Panthers gain 7 points for each touchdown and 3 points for each field goal, write a brief story of their scoring in the game based on the function.

6. An airline company creates a scatter plot showing the relationship between the number of flights an airport offers and the average distance in miles travelers must drive to reach the airport. The correlation coefficient of the relationship between the two variables is -0.52.

   a. Describe the direction and strength of the correlation.

   b. Do either of the variables cause the other to change? Explain your reasoning.

   (From Unit 4)

7. Here are clues for a puzzle involving two numbers.

   - Seven times the first number plus six times the second number equals 31.
   - Three times the first number minus ten times the second number is 29.

   What are the two numbers? Explain or show your reasoning.

   (From Unit 2)

8. Kiran shops for books during a 20% off sale.[2]

   a. What percent of the original price of a book does Kiran pay during the sale?

   b. Complete the table to show how much Kiran pays for books during the sale.

Original price in dollars, *p*	1	2	3	4	5	6
Sale price in dollars, *s*						

   c. Write an equation that relates the sale price, *s*, to the original price *p*.

   (Addressing NC.6.EE.9)

[2] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

Lesson 2: Function Notation

PREPARATION

Lesson Goals	Learning Targets
• Interpret statements that use function notation and explain (orally and in writing) their meaning in terms of a situation. • Understand that function notation is a succinct way to name a function and specify its input and output. • Use function notation to express functions with specific inputs and outputs.	• When given a statement written in function notation, I can explain what it means in terms of a situation. • I understand what function notation is and why it exists. • I can use function notation to express functions that have specific inputs and outputs.

Lesson Narrative

This lesson introduces students to function notation. Students encounter situations in which referencing certain functions and their input-output pairs gets complicated, wordy, or unclear. This motivates a way to talk about functions that is more concise and precise.

Students learn that **function notation** is a succinct way to name a function and to specify its input and output. They interpret function notation in terms of the quantities in a situation and use function notation to represent simple statements about a function. The work in this lesson prompts students to reason quantitatively and abstractly (MP2) and communicate precisely (MP6).

What are you excited for your students to be able to do after this lesson?

Focus and Coherence

Building On	Addressing
NC.8.F.1: Understand that a function is a rule that assigns to each input exactly one output. • Recognize functions when graphed as the set of ordered pairs consisting of an input and exactly one corresponding output. • Recognize functions given a table of values or a set of ordered pairs.	**NC.M1.A-REI.10:** Understand that the graph of a two variable equation represents the set of all solutions to the equation. **NC.M1.F-IF.1:** Build an understanding that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range by recognizing that: • if f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. • the graph of f is the graph of the equation $y = f(x)$. (*continued*)

	NC.M1.F-IF.2: Use function notation to evaluate linear, quadratic, and exponential functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **NC.M1.F-IF.4:** Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; and maximums and minimums.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*10 minutes*)
- **Activity 1** (*20 minutes*)
- **Activity 2** (*Optional, 10 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U5.L2 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Building On: NC.8.F.1	**Addressing:** NC.M1.A-REI.10

The purpose of this bridge is for students to identify and interpret input and output pairs within a context. Encourage students to attend to precision (MP6) by including units such as million tons. Students may be interested in researching if these numbers have increased or decreased over time.

Student Task Statement

The following table shows the amount of garbage that was produced in the US each year between 2002 and 2010 as reported by the Environmental Protection Agency (EPA).[1]

t **(years)**	2002	2003	2004	2005	2006	2007	2008	2009	2010
G **(million tons)**	239	242	249	254	251	255	251	244	250

Let G be a function which assigns to an input t (a year between 2002 and 2010) the total amount of garbage produced in that year (in million tons). Use the table to answer each of the following questions.

1. How much garbage was produced in 2004?
2. In which year did the US produce 251 million tons of garbage?
3. If the data in the table were graphed, what would the point $(2010, 250)$ represent in this context?

[1] Adapted from https://tasks.illustrativemathematics.org/

DO THE MATH	PLANNING NOTES

Warm-up: Back to the Post! (*10 minutes*)

Instructional Routine: Discussion Supports (MLR8)	
Addressing: NC.M1.A.REI.10; NC.M1.F-IF.4	**Building Towards:** NC.M1.F-IF.1

The goal of this warm-up is to motivate the need for a notation that can be used to communicate about functions.

Students analyze three graphs from an earlier lesson, interpret various points on the graphs, and use their analyses to answer questions about the situations. This work requires students to make careful connections between points on the graphs, pairs of input and output values, and verbal descriptions of the functions. Students find that, unless each feature and the function being referenced are clearly articulated, which could be tedious to do, what they wish to communicate about the functions may be ambiguous or unclear.

Step 1

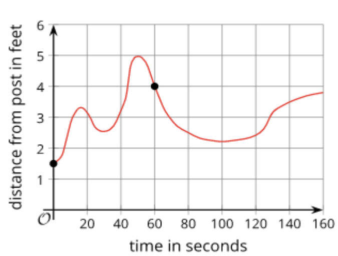
- Ask students to arrange themselves into small groups or use visibly random grouping.
- Give students a couple of minutes to answer question 1 and then time to discuss their responses with their group before moving on to the last two questions.
- When answering the last two questions, students are likely to find the prompts lacking in specificity and to probe: “for which day?” Suggest that they answer based on their interpretation of the questions.

> **Monitoring Tip:** Look for students who assume that the questions refer to one particular function and those who assume they refer to all three functions (and consequently answer them for each function). Let them know that they may be asked to share later. Include at least one student who does not typically volunteer.

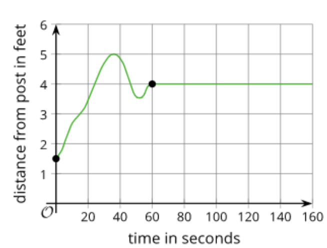
Student Task Statement

Here are the graphs of some situations you saw before. Each graph represents the distance of a dog from a post as a function of time since the dog owner left to purchase something from a store. Distance is measured in feet and time is measured in seconds.

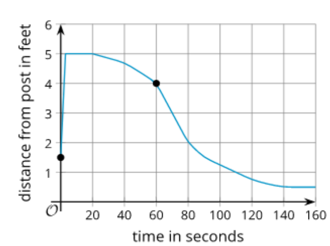
Day 1



Day 2



Day 3



1. Use the given graphs to answer these questions about each of the three days:

   a. How far away was the dog from the post 60 seconds after the owner left?
   Day 1: Day 2: Day 3:

   b. How far away was the dog from the post when the owner left?
   Day 1: Day 2: Day 3:

   c. The owner returned 160 seconds after he left. How far away was the dog from the post at that time?
   Day 1: Day 2: Day 3:

   d. How many seconds passed before the dog reached the farthest point it could reach from the post?
   Day 1: Day 2: Day 3:

2. Consider the statement, “The dog was 2 feet away from the post after 80 seconds.” Do you agree with the statement?

3. What was the distance of the dog from the post 100 seconds after the owner left?

Step 2

- Invite students to share their response to the first set of questions.
  - To help illustrate that it could be tedious to refer to a specific part of a function fully and precisely, ask each question completely for each of the three days. (For instance, “How far away was the dog from the post 60 seconds after the owner left on Day 1? How far away was the dog from the post 60 seconds after the owner left on Day 2?”) If students offer a numerical value (for instance, “1.5 feet”) without stating what question it answers or to what quantity it corresponds to, use *Discussion Supports* to ask them to clarify and display the sentence frame, “The dog was ___ feet away from the post ___ seconds after the owner left on Day ___.”
- Next, select previously identified students to share their responses to the last two questions. Regardless of whether students chose to answer them for a particular day or for all three days, point out that the answers depend on the day. When the day (or the function) is not specified, it is unclear what information is sought.
- Explain that sometimes we need to be pretty specific when talking about functions, but to be specific could require many words and become burdensome. Tell students that they will learn about a way to describe functions clearly and succinctly.

DISCUSSION SUPPORTS	***What Is This Routine?*** The teacher uses multi-modal strategies for helping students comprehend and generate language and ideas, such as sentence frames, word walls, images and videos, revoicing, choral response, gesture, and graphic organizers. The strategies can be combined and used together with any of the other routines. ***Why This Routine?*** *Discussion Supports* (MLR8) foster rich and inclusive discussions about mathematical ideas, representations, contexts, and strategies. Use *Discussion Supports* to make classroom communication accessible, to increase meta-awareness of language, and to demonstrate strategies students can use to enhance their own communication and construction of ideas.

DO THE MATH	PLANNING NOTES

Activity 1: A Handy Notation (*20 minutes*)

Instructional Routine: Critique, Correct, Clarify (MLR3)	
Addressing: NC.M1.F-IF.1; NC.M1.F-IF.2	

In this activity, students learn that function notation can be used as a handy shorthand for communicating about functions and specific parts or features of a function. They interpret statements that are written in this notation and use the notation to refer to points on a graph or to represent simple verbal statements about a function.

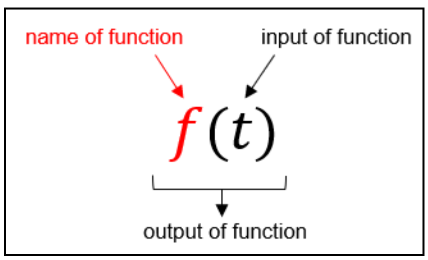
RESPONSIVE STRATEGIES

Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of function notation. Provide students with a physical copy of the annotated function notation.

Supports accessibility for: Conceptual processing; Language

Step 1

- Explain to students that one way to talk about functions precisely and without wordy descriptions is by naming the functions and using function notation.
  - Suppose we give a name to each function that relates the dog's distance from the post and the time since the dog owner left: function f for Day 1, function g for Day 2, function h for Day 3. The input of each function is time in seconds, t.
  - To make sure students see the structure of this new notation, consider displaying it and annotating each part, as shown here.



- Clarify that:
  - The notation $f(t)$ is read "f of t." It tells us that f is the name of the function, t is the input of the function, and $f(t)$ is the output or the value of the function when the input is t. For example, $f(100)$ refers to the distance of the dog from the post on Day 1, after 100 seconds.
  - The statement $g(t) = d$ is read: "g of t is equal to d." It tells us that g is the name of the function and t is the input. It also tells us that $g(t)$ is the output or the value of the function at t, and that output value is d. The statement $g(100) = 4$ means that on Day 2, after 100 seconds, the dog was 4 feet from the post.
  - To represent "the distance of the dog from the post 60 seconds after the owner left," we can simply write: $f(60)$. To express the same quantity for the second and third day, we can write $g(60)$ and $h(60)$.
  - Explain that the parentheses are being used differently than what students have seen in the past. The parentheses here do not mean to multiply. The parentheses are used to indicate the input value. The letter being used is not a variable but the name of the function.

Step 2

- Keep students in their groups.
- Ask students to refer to the three graphs from the warm-up to answer question 1 and 2a. Tell students they will have an opportunity to discuss 2a before completing the rest of the question.
- Give students 1–2 minutes of quiet work time then a few more minutes to discuss their thinking with their group.

Advancing Student Thinking**:** Students may ignore the function name and attend only to the input value. For instance, they may say "$f(60)$ means that 60 seconds have passed." Explain that the input value of 60 or $t = 60$ does represents that 60 seconds have passed, but the expression $f(60)$ represents the "output value" of the function. In this case, it means the dog's distance from the post, on Day 1, 60 seconds after its owner left.

Student Task Statement

Let's name the functions that relate the dog's distance from the post and the time since its owner left: function f for Day 1, function g for Day 2, function h for Day 3. The input of each function is time in seconds, t.

1. Use function notation to complete the table.

	Day 1	Day 2	Day 3
a. distance from post 60 seconds after the owner left	$f(60)$		
b. distance from post when the owner left			
c. distance from post 150 seconds after the owner left			

2. Describe what each expression represents in this context:

   a. $f(15)$ b. $g(48)$ c. $h(t)$

3. The equation $g(120) = 4$ can be interpreted to mean: "On Day 2, 120 seconds after the dog owner left, the dog was 4 feet from the post." What does each equation mean in this situation?

   a. $h(40) = 4.6$ b. $f(t) = 5$ c. $g(t) = d$

Step 3

- Facilitate a whole-group discussion. Begin by inviting students to share their responses to question 1.
  - As students begin to share, they may be unsure as to how to express the notation orally. Explain that the expression $f(60)$ is read "f of 60," $g(150)$ is read "g of 150," and $h(t)$ is read "h of t."

RESPONSIVE STRATEGY

Create an anchor chart for function notation. For example, f(x) is read as "f of x."

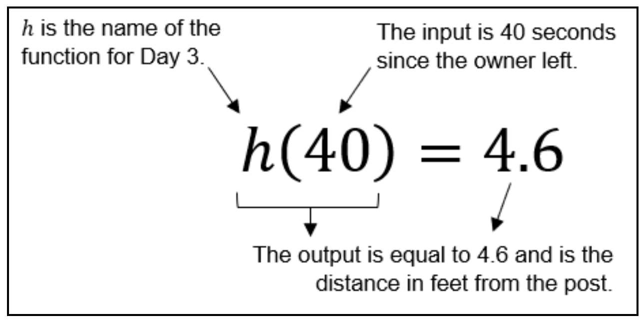
- Before students share their description of what the expression in question 2a represents in context, use the *Critique, Correct, Clarify* routine by providing students with the following first draft writing to improve: "$f(15)$ is the input value. It tells us that 15 seconds have passed."
  - Give students 1 minute to first identify any parts of the first draft that are unclear, incomplete, or incorrect. Display the first draft writing and spend 1–2 minutes having two or three students share ideas about what parts need improvement. As students share, annotate the displayed writing by circling, underlining, and marking it with arrows and labels (e.g., "clarify," "add detail," "?" etc.).
  - Ask students (individually or in pairs) to spend 1–2 minutes writing a second draft that is more clear, complete, and correct than the first draft. Circulate and look for students who clarify that $f(15)$ represents an output value, specifically the distance of the dog from the post 15 seconds after the owner left on Day 1. Prioritize student work that still has some ambiguity in the language, in order to allow for collaborative editing as a whole class.

- Invite one or two students to read their second drafts aloud. Spend 3–5 minutes engaging in “live editing” to generate a third draft by scribing slowly as each student reads their draft aloud. While scribing is happening, invite the author of each draft, and the rest of the class as well, to offer revised wording and additional details. This helps students evaluate, and improve upon, the written mathematical arguments of others, as they interpret function notation in context.

CRITIQUE, CORRECT, CLARIFY	***What Is This Routine?*** Students are given a piece of mathematical writing that is not their own to analyze, reflect on, and develop. The writing is an incorrect, incomplete, or unclear “first draft” written argument or explanation, and the students’ job is to improve the writing by correcting any errors, clarifying meaning, and adding explanation, justification, or details. The routine begins with a brief critique of the first draft in which the teacher elicits two or three ideas from students to identify what could use improvement; students then individually write second drafts, and finally, the teacher scribes as two or three students read their second drafts aloud. ***Why This Routine?*** *Critique, Correct, Clarify* (MLR3) prompts student reflection, fortifies output, and builds students’ meta-linguistic awareness. The final step of public scribing creates an opportunity to invite all students to help edit a final draft together, so that it makes sense to more people. Teachers can demonstrate with meta-think-alouds and press for details when necessary.

Step 4

- Give students 2–3 minutes to complete the rest of the activity and discuss within their group.
- Invite students to share their responses to question 3.
- Display the equations for all to see and annotate as students share their responses. Ask students questions to connect their response to the given equation. For example, $h(40) = 4.6$ means that on Day 3, 40 seconds after the owner left, the dog is 4.6 feet from the post.



  - “How did you determine the day from the given equation?” (h is the name for the Day 3 function.)
  - “How did you determine what the 40 meant in this situation?” (The 40 is the input, which is the time since the owner left.)
  - “How did you determine what the 4.6 meant in this situation?” (The output $h(40)$ is 4.6, which is the distance in feet from the post.)
- If necessary, remind students that values that are not known can still be interpreted, e.g. “$f(t) = 5$ means that on Day 1, t seconds after the owner left, the dog is 5 feet from the post.”

DO THE MATH	PLANNING NOTES

Activity 2: Birthdays (*Optional, 10 minutes*)

Instructional Routine: Collect and Display (MLR2) - Responsive Strategy	
Addressing: NC.M1.F-IF.1; NC.F-IF.2	

This optional activity reinforces students' understanding about what makes a relationship between two variables a function, namely, that it gives a unique output for each input. It also prompts students to use function notation to express a functional relationship that does not involve numerical values for its input and output.

Step 1

- Keep students in groups.
- Ask each group to work together to complete questions 1–5.

Advancing Student Thinking: If a student wonders what happens to a person born on February 29, tell them that the output of the function is the original birth date, not the annual birthday.

RESPONSIVE STRATEGY

As students work, listen for and collect vocabulary words, everyday phrases, and diagrams students use to describe what makes a relationship between two variables a function. Capture student language that reflects a variety of ways to describe the characteristics of and differences between a relationship that is a function and a relationship that is not a function. Listen for and revoice the words "unique," "input," and "output." Write the students' words on a visual display and update it throughout the remainder of the lesson. Remind students to borrow language from the display as needed. This will help students read and use mathematical language during their partner and whole-group discussions.

Collect and Display (MLR2)

Student Task Statement

Rule B takes a person's name as its input and gives their birthday as the output.

Rule P takes a date as its input and gives a person with that birthday as the output.

Rule B

Input	Output
Abraham Lincoln	February 12

Rule P

Input	Output
August 26	Katherine Johnson

1. Complete each table with three more examples of input-output pairs.
2. If you use your name as the input to B, how many outputs are possible? Explain how you know.
3. If you use your birthday as the input to P, how many outputs are possible? Explain how you know.
4. Only one of the two relationships is a function. The other is not a function. Which one is which? Explain how you know.
5. For the relationship that is a function, write two input-output pairs from the table using function notation.

Are You Ready For More?

1. Write a rule that describes these input-output pairs:

   $F(ONE) = 3$ $F(TWO) = 3$ $F(THREE) = 5$ $F(FOUR) = 4$

2. Here are some input-output pairs with the same inputs but different outputs:

   $v(ONE) = 2$ $v(TWO) = 1$ $v(THREE) = 2$ $v(FOUR) = 2$

   What rule could define function v?

Step 2

- Discuss with students:
  - "Why is B a function, but P isn't?" (Each input for B has a unique output, while inputs for P may have several outputs. For example, March 14 is the birthday of Albert Einstein, Stephen Curry, Billy Crystal, Simone Biles, and many other people. February 12 is the birthday of Abraham Lincoln and Charles Darwin.)
  - "Would it be acceptable to express relationship P using function notation, for instance,: $P(August\ 26) = Katherine\ Johnson$? Why or why not?" (No, because this notation is reserved for functions.)
- Some students might wonder if B is still a function if multiple people have the same name. For instance, there might be a few people named Katherine Johnson, and if we enter "Katherine Johnson" as the input for B, we would likely get different birthdays for the output.
  - Acknowledge that this is true, and that B would only be a function if it assumes that no two people have the same full name, or if another identifier could be used to tell apart people with the same first name and last name (for instance, if a middle name or initial is also used, or if a number is added to each Katherine Johnson to distinguish them from one another).

COLLECT AND DISPLAY	***What Is This Routine?*** The teacher captures students' oral words and phrases into a stable, collective reference in order to stabilize the fleeting language that students use during partner, small-group, or whole-class activities. The teacher listens for, and scribes, the student output using written words, diagrams, and pictures; this collected output can be organized, revoiced, or explicitly connected to other language in a display for all students to use over the course of a lesson or unit. ***Why This Routine?*** *Collect and Display* (MLR2) provides feedback for students in a way that increases accessibility while simultaneously supporting meta-awareness of language. The routine mirrors student language back to the whole class to enable students' own output to be used as a reference in developing their mathematical language over time.

DO THE MATH	PLANNING NOTES

Lesson Debrief (*5 minutes*)

Instructional Routine: Take Turns

The purpose of this lesson is for students to use and interpret function notation. Function notation includes the name of the function, the input value and the output value in a succinct and compact form. It can be interpreted to describe contextual situations.

Engage students in the *Take Turns* routine as part of the debrief of this lesson. Refer back to the bagel shop activity from Lesson 1. Display the following for all to see:

> The best price for bagels, in dollars, is a function of the number of bagels bought, n.
>
> a. $b(2)$
> b. $b(6)$
> c. $b(11) = 10.50$
> d. $b(13) = 11.25$
> e. $b(x) = 8.50$

- Have students arrange themselves in pairs or use visibly random grouping.
- Ask partners to *Take Turns* reading and interpreting the statements in function notation. Each person should:
  - Read the statement aloud to their partner.
  - Identify the input, the output, and the function in the statement.
  - Explain the meaning of the entire statement using a complete sentence.
- If students say that the first two statements have no outputs, clarify that both $b(2)$ and $b(6)$ represent outputs, even though the value of each is not stated.

PLANNING NOTES

TAKE TURNS

What Is This Routine? Students work with a partner or small group. They take turns in the work of the activity, whether it be spotting matches, explaining, justifying, agreeing or disagreeing, or asking clarifying questions. If they disagree, they are expected to support their case and listen to their partner's arguments. The first few times students engage in these activities, the teacher should demonstrate, with a partner, how the discussion is expected to go. Once students are familiar with these structures, less set-up will be necessary. While students are working, the teacher can ask students to restate their question more clearly or paraphrase what their partner said.

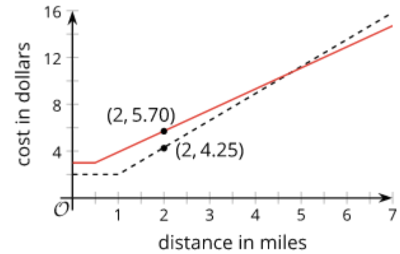
Why This Routine? Building in an expectation, through the *Take Turns* routine, that students explain the rationale for their choices and listen to another's rationale deepens the understanding that can be achieved through these activities. Specifying that students take turns deciding, explaining, and listening limits the phenomenon where one student takes over and the other does not participate. Taking turns can also give students more opportunities to construct logical arguments and critique others' reasoning (MP3).

Student Lesson Summary and Glossary

Here are graphs of two functions, each representing the cost of riding in a taxi from two companies: Friendly Rides and Great Cabs.

For each taxi, the cost of a ride is a function of the distance traveled. The input is distance in miles, and the output is cost in dollars.

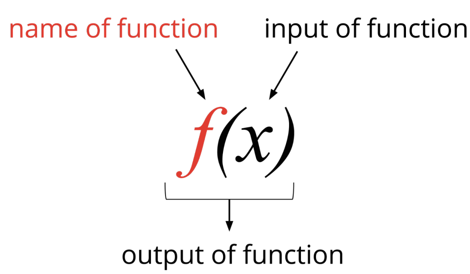
- The point $(2, 5.70)$ on one graph tells us the cost of riding a Friendly Rides taxi for 2 miles.
- The point $(2, 4.25)$ on the other graph tells us the cost of riding a Great Cabs taxi for 2 miles.



We can convey the same information much more efficiently by naming each function and using **function notation** to specify the input and the output.

- Let's name the function for Friendly Rides function f.
- Let's name the function for Great Cabs function g.
- To refer to the cost of riding each taxi for 2 miles, we can write: $f(2)$ and $g(2)$.
- To say that a 2-mile trip with Friendly Rides will cost $5.70, we can write $f(2) = 5.70$.
- To say that a 2-mile trip with Great Cabs will cost $4.25, we can write $g(2) = 4.25$.

In general, function notation has this form:



It is read "f of x" and can be interpreted to mean: $f(x)$ is the output of a function f when x is the input.

The function notation is a concise way to refer to a function and describe its input and output, which can be very useful. Throughout this unit and the course, we will use function notation to talk about functions.

Function notation: A particular way of writing the outputs of a function that you have given a name to. If the function is named f and x is an input, then $f(x)$ denotes the corresponding output.

Cool-down: A Growing Puppy (*5 minutes*)

Addressing: NC.M1.F-IF.1; NC.M1.F-IF.2

Cool-down Guidance: More Chances
Students will have more opportunities to understand the mathematical ideas in this cool-down, so there is no need to slow down or add additional work to the next lessons. Instead, use the results of this cool-down to provide guidance for what to look for and emphasize over the next several lessons to support students in advancing their current understanding.

Cool-down

Function Q gives a puppy's weight in pounds as a function of its age in months.

1. What does each expression or equation represent in this situation?
   a. $Q(18)$
   b. $Q(30) = 27.5$
2. Use function notation to represent each statement.
   a. When the puppy turned 12 months old, it weighed 19.6 pounds.
   b. When the puppy was M months old, it weighed W pounds.

Student Reflection:

What were some of your most powerful learning moments in class today? What made it powerful?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

What evidence have students given that they understand function notation? What language do they use or associate with $f(x)$?

Practice Problems

1. The height of water in a bathtub, w, is a function of time, t. Let P represent this function. Height is measured in inches and time in minutes. Match each statement in function notation with a description.

   a. $P(0)=0$
   b. $P(4)=10$
   c. $P(10)=4$
   d. $P(20)=0$

   1. After 20 minutes, the bathtub is empty.
   2. The bathtub starts out with no water.
   3. After 10 minutes, the height of the water is 4 inches.
   4. The height of the water is 10 inches after 4 minutes.

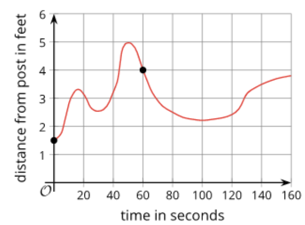
2. Function C takes time for its input and gives a student's Monday class for its output.

   a. Use function notation to represent: A student has English at 10:00.

   b. Write a statement to describe the meaning of $C(11{:}15)=\text{chemistry}$.

3. Function f gives the distance of a dog from a post, in feet, as a function of time, in seconds, since its owner left.

   Find the approximate values of $f(20)$ and of $f(140)$.



4. Function C gives the cost, in dollars, of buying n apples. What does each expression or equation represent in this situation?

   a. $C(5)=4.50$
   b. $C(2)$

5. A number of identical cups are stacked up. The number of cups in a stack and the height of the stack in centimeters are related.

   a. Can we say that the height of the stack is a function of the number of cups in the stack? Explain your reasoning.

   b. Can we say that the number of cups in a stack is a function of the height of the stack? Explain your reasoning.

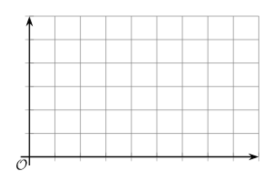
   (From Unit 5, Lesson 1)

6. In a function, the number of cups in a stack is a function of the height of the stack in centimeters.

   a. Sketch a possible graph of the function on the coordinate plane. Be sure to label the axes.

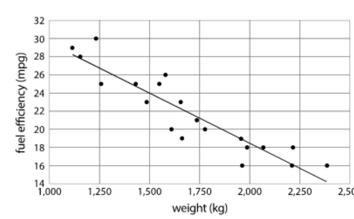
   b. Identify one point on the graph and explain the meaning of the point in the situation.

   (From Unit 5, Lesson 1)



7. The graph below describes various cars at a car dealership, and the line $y=-0.011x+40.604$ is a line of best fit. Interpret the slope and y-intercept based on the labels of the x and y axes.[2]

   (From Unit 4)



8. Solve each system of equations without graphing. Show your reasoning.

a. $\begin{cases} -5x + 3y = -8 \\ 3x - 7y = -3 \end{cases}$

b. $\begin{cases} -8x - 2y = 24 \\ 5x - 3y = 2 \end{cases}$

(From Unit 3)

9. Jada and Kiran are solving the inequality $4x - 6 > 7x + 8$. Their steps are listed below:

Jada

$$4x - 6 > 7x + 8$$
$$-3x - 6 > 8$$
$$-3x > 14$$
$$x > -\frac{14}{3}$$

Kiran

$$4x - 6 > 7x + 8$$
$$-6 > 3x + 8$$
$$-14 > 3x$$
$$-\frac{14}{3} > x$$

Do you agree with Jada, Kiran, both, or neither? Explain your reasoning.

(From Unit 2)

10. The table below represents the population of India by decade.

Year	1960	1970	1980	1990	2000	2010	2020
Population (in millions)	451	555	699	873	1,057	1,234	1,380

Let P be a function which assigns to an input t (a year between 1960 and 2020) the population in that year (in millions of people). Use the table to answer each of the following questions.

a. What was the population of India in the year 2000?

b. In which year was the population about 700 million people?

c. If the data in the table were graphed, what would the point $(2020, 1380)$ represent?

(Addressing NC.8.F.1)

Lesson 3: Interpreting and Using Function Notation

PREPARATION

Lesson Goals	Learning Targets
• Describe connections between statements that use function notation and a graph of the function. • Practice interpreting statements that use function notation and explaining (orally and in writing) their meaning in terms of a situation. • Sketch a graph of a function given statements in function notation.	• I can describe the connections between a statement in function notation and the graph of the function. • I can use function notation to efficiently represent a relationship between two quantities in a situation. • I can use statements in function notation to sketch a graph of a function.

Lesson Narrative

In this lesson, students continue to develop their ability to interpret statements in function notation in terms of a situation, including reasoning about inequalities such as $f(a) > f(b)$. They now have to pay closer attention to the units in which the quantities are measured to effectively interpret symbolic statements. Along the way, students practice reasoning quantitatively and abstractly (MP2) and attending to precision (MP6).

Students also begin to connect statements in function notation to graphs of functions. They see each input-output pair of a function f as a point with coordinates $(x, f(x))$ when x is the input, and they use information in function notation to sketch a possible graph of a function.

Students' work with graphs is expected to be informal here. In a later lesson, students will focus on identifying features of graphs more formally.

What math language will you want to support your students with in this lesson? How will you do that?

Focus and Coherence

Building On	Addressing
NC.8.F.5: Qualitatively analyze the functional relationship between two quantities. • Analyze a graph determining where the function is increasing or decreasing; linear or non-linear. • Sketch a graph that exhibits the qualitative features of a real-world function.	**NC.M1.A-REI.10:** Understand that the graph of a two variable equation represents the set of all solutions to the equation. **NC.M1.F-IF.2:** Use function notation to evaluate linear, quadratic, and exponential functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **NC.M1.F-IF.4:** Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; and maximums and minimums.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*15 minutes*)
- **Activity 2** (*10 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U5.L3 Cool-down (print 1 copy per student)

LESSON

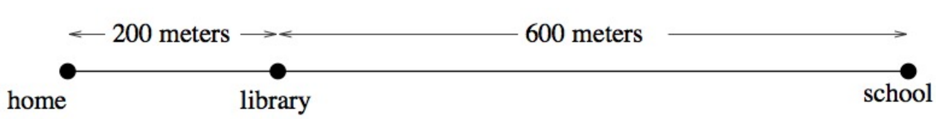
Bridge (*Optional, 5 minutes*)

Building On: NC.8.F.5

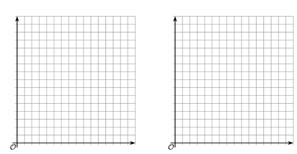
The purpose of this bridge is for students to sketch a graph based on the information and context provided. Students use the context to identify important features of the graph such as initial distance from the school, intervals when the function is increasing, and intervals when the function is decreasing. In the lesson, students will use the context and function notation to identify important features and use them to sketch a graph. This task is aligned to question 8 in Check Your Readiness.

Student Task Statement

Nina rides her bike from her home to school, passing by the library on the way and traveling at a constant speed for the entire trip.[1] (See map below.)



1. Sketch a graph of Nina's distance from school as a function of time.
2. Sketch a graph of Nina's distance from the library as a function of time.



[1] Adapted from https://tasks.illustrativemathematics.org/

DO THE MATH

PLANNING NOTES

Warm-up: Observing a Drone (*5 minutes*)

Addressing: NC.M1.F-IF.2

In this warm-up, students are prompted to compare function values. To do so, they need to interpret statements in function notation and connect their interpretations to the graph of the function. As students work, they will likely notice and use informal language to express that the function is increasing at first, then remaining constant, then decreasing. In Step 2, students are introduced to the meaning of $f(0)$ and asked to identify the moments when the drone levels off, starts decreasing and lands. This previews several key features of a function, which will become formalized in Lesson 6.

Previously, students recognized that if a point with coordinates $(2, 20)$ is on the graph of a function, the 2 is an input and the 20 is a corresponding output. Here, they begin to see the second value in a coordinate pair more abstractly, as $f(2)$ when the input is 2, or as $f(4)$ when the input is 4.

RESPONSIVE STRATEGY

Some students may benefit from writing in some reasonable heights for the y-axis. For example, 5, 10, 15, 20.

Step 1

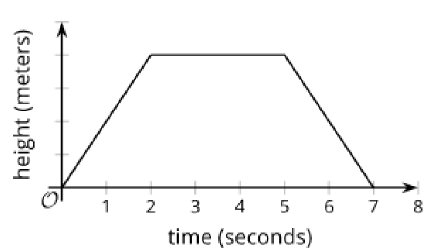
- Give students 1–2 minutes of quiet think time.
- Some students might be unfamiliar with drones. If needed, give a brief explanation of what they are.

Student Task Statement

Here is a graph that represents function f, which gives the height of a drone, in meters, t seconds after it leaves the ground.

Decide which function value is greater, if they are equal, or if we can't tell. Explain your reasoning.

1. $f(0)$ or $f(4)$
2. $f(2)$ or $f(5)$
3. $f(3)$ or $f(7)$
4. $f(t)$ or $f(t+1)$



Step 2

- Invite students to share how they compared each pair of outputs. If not mentioned in students' explanations, point out that for each pair, we are comparing the vertical values of two points on the graph. Even though the statements don't tell us the values of, say, $f(3)$ and $f(7)$, and the vertical axis shows no scale, we can tell from the graph that the function has a greater value when t is 3 than when t is 7.

Advancing Student Thinking: Students may confuse comparisons for $f(3)$ and $f(7)$ as comparing 3 and 7 with respect to the x-axis. If students have this misconception, prompt the class to vote for agree or disagree, elicit explanations from both opinions, then ask if anyone was convinced to change their original vote. This allows students to make sense of the mathematics, as the authority, rather than finding out their thinking was "wrong."

- Tell students that the coordinates $(0, f(0))$ represent the starting point of the drone. Ask students, "What are the coordinates of the points when the drone starts leveling off? when it starts to descend? when it lands?"

  ($(2, f(2)), (5, f(5))$, and $(7, f(7))$, respectively)

- Highlight that the coordinates of each point on a graph of a function are $(x, f(x))$.

- Discuss with students how they compared $f(t)$ and $f(t+1)$ (in the last question). Make sure students see that the $f(t+1)$ could be less than, equal to, or greater than $f(t)$, depending on the value of t.
  - If t is 2, then $t+1$ is 3. From the graph, we can see that $f(2)$ is equal to $f(3)$ because the graph has the same vertical value when t is 2 and 3.
  - If t is 5, then $t+1$ is 6. From the graph, we can see that $f(5)$ is greater than $f(6)$ because the graph has a greater vertical value when t is 5 than when t is 6.

DO THE MATH	PLANNING NOTES

Activity 1: Smartphones (*15 minutes*)

Instructional Routines: Discussion Supports (MLR8) - Responsive Strategy; Compare and Connect (MLR7)
Addressing: NC.M1.A-REI.10; NC.M1.F-IF.2; NC.M1.F-IF.4

In this activity, students continue to interpret statements in function notation in terms of a situation. Several things are new here, all of which provide opportunities to attend to precision (MP6) and to reason quantitatively and abstractly (MP2).

- The output of the function is measured in millions, so students need to attend carefully to the units or otherwise may misinterpret the situation (for example, saying that about 2 thousand people owned smartphones in 2017 rather than about 2 billion people).
- The input of the function is measured in "years after 2000," so a positive input value t needs to be interpreted as year $2000+t$, and the year 2010 to be associated with $t=10$.
- One of the input values is negative, so students will need to reason about what this means in this situation.

Students also consolidate various pieces of information about the function, given in descriptions and function notation, in order to sketch the graph of a function. It is possible for students to sketch many different graphs through the given points, but the context suggests that, over time, the function values are increasing roughly exponentially (though students do not need to know that term at this point).

Step 1

- Read the opening sentence in the activity statement as a class. Ask students to identify the input and output of this function, and the units in which each variable is measured.
- Ask students to arrange themselves into pairs or use visibly random grouping.
- Give students 1–2 minutes of quiet time to read and make sense of the first two questions (without writing responses). Urge them to think about what the input and output values are in each statement (given in function notation or in words).
- Give students another few minutes to share their thinking with their partner, and then, when reaching an agreement, to write their responses. Prompt students to write their interpretations for statements such as $P(17) = 2,320$ in complete sentences and to use the quantity names and units.

RESPONSIVE STRATEGIES

Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their strategy. For example, "____ stands for ____.", "What does this part of ____ mean?", "What other details are important?", and "___ million people owned a smartphone in the year ___."

Supports accessibility for: Language; Social-emotional skills

Discussion Supports (MLR8)

Advancing Student Thinking: Some students may struggle to express a number like 296,600,000 in millions, or they may think that a quantity like "2,320 million" doesn't quite make sense. Ask them to write these quantities as numerals: 1 million, 10 million, 100 million, and so on, and then use their list to help figure out how to say 296,600,000 in millions or to write 2,320 million as a numeral.

Step 2

- Facilitate a whole-class discussion. The purpose of this discussion is for students to make sense of the situation and understand the units used for years since 2000 and population in millions.
- Invite students to share their responses to the first two questions. Before students complete the remaining questions, make sure they see why:
  - The number of smartphone owners in the first question is *not* a couple of thousand people. The output is measured in millions, so a number such as 2,320 means 2,320 million or 2.32 billion people.
  - The input value for the year 2010 is *not* 2010. The input is measured in "years after the year 2000," so the input for the year 2010 is 10. An input of 2010 would mean the year 4010.
- Ask students to continue working in pairs to complete questions 3 and 4.

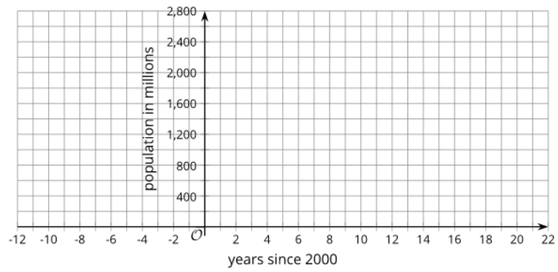
Monitoring Tip: As students work, look for different sketches of the graph. Students may graph the points $(-10, 0)$, $(10, 296.6)$, $(15, 1860)$, and $(17, 2320)$ but have different ways of sketching what occurs between the points, such as: connecting points with straight lines, drawing a curve passing through the points, or leaving only the points. Identify students who (1) graph with just the points; (2) graph with line segments connecting the points; (3) graph with points and a modeling line; (4) graph with points and a curve; and (5) graph in another way, and let them know that they may be asked to share later. Include at least one student who does not typically volunteer.

Advancing Student Thinking: Students may think they do not have enough information to sketch a graph of P. Encourage them to read through the activity to identify points that must be part of the graph of P, but not written in (x, y) form.

Student Task Statement

The function P gives the number of people, in millions, who own a smartphone t years after the year 2000.

1. What does each equation tell us about smartphone ownership?
   a. $P(17) = 2,320$
   b. $P(-10) = 0$
2. Use function notation to represent each statement.
   a. In 2010, the number of people who owned a smartphone was 296,600,000.
   b. In 2015, about 1.86 billion people owned a smartphone.
3. Mai is curious about the value of t in $P(t) = 1,000$.
   a. What would the value of t tell Mai about the situation?
   b. Is 4 a possible value of t here?
4. Use the information you have so far to sketch a graph of the function.



Are You Ready For More?

What can you say about the value or values of t when $P(t) = 1,000$?

Part 3

- Select students to share their responses to question 3.
  - Make sure students can interpret statements such as $P(15) = 1,860$ and $P(t) = 1,000$ in terms of the situation and articulate them completely. For instance, students might say, “The output is 1,860 when the input is 15,” or “When the input is t years, the output is 1,000 million,” or some other variation that doesn’t convey the quantities fully. Push students to refine their interpretation so that it is clear that $P(15) = 1,860$ means “1.86 billion people owned a smartphone in the year 2015” and $P(t) = 1,000$ means “A billion people owned a smartphone t years after 2000.”
- Use the *Compare and Connect* routine to select previously identified students to share their sketches of the graph of P.
  - Ask students, “What is similar between these graphs? What is different?” (They all have the four given points, but what is happening between the points is different.)
  - Acknowledge that a graph of P could be drawn in various ways. The information we have is limited to the four input-output pairs, so what happens between the points is up for interpretation. But it would make sense, based on the context, for the graph to show very little change before the year 2010 and then a rapid increase afterward.

- If possible, draw students' attention to the idea that modeling a relationship with a function involves making choices about the units used. Discuss questions such as:
    - "The output of P is defined in terms of 'millions of people' instead of individual persons. What might be a reason for this?" (If the unit is "persons," the scale would show very long numbers with 9 or 10 digits each, which would be much harder to read and might lead to mistakes. Using "in thousands" or "in millions" strategically makes it possible to use simpler numbers. It makes it easier to draw attention to important characteristics of a graph.)
    - "The input is defined in 'years after 2000.' Could we have instead used calendar years, such as 2002?" (Yes.)
    - "What might be a reason to choose 'years after 2000' as the unit?" (It allows us to write smaller numbers. Another reason is that sometimes knowing a duration is more useful than knowing a particular point in time at which something happens. For instance, in the drone activity, knowing the number of seconds that had passed before the drone landed was more useful than knowing that it landed at, say, 2:45 p.m. In this activity, the year 2000 might be significant in the development of smartphones, so measuring time with that starting point might be useful.)

COMPARE AND CONNECT	***What Is This Routine?*** The teacher facilitates a discussion about two or more pieces of student work that include distinct mathematical representations or approaches to a problem, calling attention to the correspondences among quantities, relationships, and features of the representations. Teachers should demonstrate thinking out loud (e.g., exploring why one might do or say it this way, questioning an idea, wondering how an idea compares or connects to other ideas or language), and students should be prompted to reflect and respond. ***Why This Routine?*** Use *Compare and Connect* (MLR7) to foster students' meta-awareness as they identify, compare, and contrast different mathematical approaches, representations, and language. This routine supports meta-cognitive and meta-linguistic awareness, and also supports mathematical conversation.

DO THE MATH	PLANNING NOTES

Activity 2: Boiling Water (*10 minutes*)

Instructional Routines: Take Turns; Compare and Connect (MLR7); Discussion Supports (MLR8)
Addressing: NC.M1.A-REI.10; NC.M1.F-IF.2; NC.M1.F-IF.4

Previously, students learned that each point on the graph of a function f is of the form $(t, f(t))$ for input t and corresponding output $f(t)$. They analyzed and plotted input-output pairs in which both values were known.

In this activity, students reason about unknown output values by relating them to values that are known (by interpreting inequalities such as $W(5) > W(2)$ and equations such as $W(30) - W(20) = -75$), using a graph and a context to support their reasoning. The work here prompts students to reason quantitatively and abstractly (MP2).

Students *Take Turns* explaining their interpretation of statements written in function notation and making sense of their partner's interpretation, discussing their differences if they disagree. In so doing, students practice constructing logical arguments and critiquing the reasoning of others (MP3).

Step 1

- Keep students in pairs.
- Give students a few minutes of quiet time to think about the meaning of each statement in the first question.
- Then, give students time to take turns sharing their explanations with their partner.
  - Remind students that when one student explains, the partner's job is to listen and make sure that they agree and that the explanation makes sense. If they don't agree, the partners discuss until they come to an agreement.
- Based on their shared interpretation of the statements, partners then sketch their own graph of the function.

RESPONSIVE STRATEGY

Use color coding and annotations to highlight connections between representations in a problem. For example, invite students to highlight the W, 0, and 72 in W(0)=72 in different colors and highlight the corresponding parts of the first sentence of the task statement in the same color to show what each part represents. Remind students to refer to their copy of the annotated function notation.

Supports accessibility for: Visual-spatial processing

Monitoring Tip: Identify students who can correctly interpret statements in terms of the situation and in relation to the graph of the function. Also, look for students whose graphs are very different but are both correct. Let them know that they may be asked to share later. Include at least one student who does not typically volunteer.

Advancing Student Thinking: Some students may think there is not enough information to accurately graph the function. Assure them that this is true, but clarify that we are not after a specific graph but rather a possible graph of the function based on the information we do have.

Student Task Statement

The function W gives the temperature, in degrees Fahrenheit, of a pot of water on a stove t minutes after the stove is turned on.

1. Take turns with your partner to explain the meaning of each statement in this situation. When it's your partner's turn, listen carefully to their interpretation. If you disagree, discuss your thinking and work to reach an agreement.

   a. $W(0) = 72$

   b. $W(5) > W(2)$

   c. $W(10) = 212$

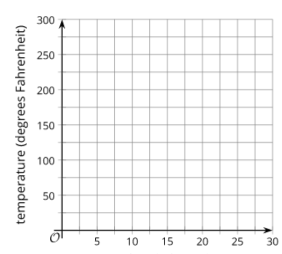
   d. $W(12) = W(10)$

   e. $W(15) > W(30)$

   f. $W(30) - W(20) = -75$

2. If all statements in the previous question represent the situation, sketch a possible graph of function W.

   Be prepared to show where each statement can be seen on your graph.



Are You Ready For More?

A family went on a hiking trip, and the temperature changed drastically as they traveled up the mountain. The function C gives the temperature, in degrees Celsius, of the air t minutes after the family started hiking. Explain the meaning of each statement in this situation.

a. $C(0) = 11$

b. $C(45) > C(90)$

c. $C(120) = -3$

d. When $100 < t < 130$, $C(t) < 0$. Otherwise, $C(t) \geq 0$.

e. Draw a graph that could represent this function.

Step 2

- Facilitate a whole-class discussion. The purpose of the discussion is to relate the information interpreted from the function notation to the important features of the graph. These include the initial temperature of 72°F, the temperature increasing between 2 and 5 minutes, the temperature of 212°F at 10 and 12 minutes, the temperature decreasing between 15 and 30 minutes, and a decrease of 25°F between 20 and 30 minutes.

Use the *Compare and Connect* routine by selecting previously identified students to share their interpretations of the inequalities and equations in the first question and to show their graphs. Ask these students to explain how each statement is evident in their graph, and then invite the class to identify what is the same and what is different about how each pair approached the graphs as a means for solidifying language and conceptual understanding at the same time.

Use *Discussion Supports* to support whole-class discussion. After each student shares, provide the class with the following sentence frames to help them respond: "I agree because..." or "I disagree because…" If necessary, re-voice students' ideas by restating their statements as questions. For example, if a student says, "the temperature was 72 degrees Fahrenheit," ask: "at what time (or input value) was the temperature 72 degrees Fahrenheit?" This will help students listen and respond to each other as they explain the meaning of each statement represented in function notation.

- If time permits, discuss questions such as:
  - "Why might it be true that $W(15) > W(30)$?" (The heat was turned off at or after 15 minutes, or the kettle was taken off the stove.)
  - "You and your partner agreed on what each statement meant. Are your graphs identical? If not, why might that be?" (Function W was not completely defined. We have information about the temperature at some points in time and how they compare, but we don't have all the information about all points in time.)

DO THE MATH

PLANNING NOTES

Lesson Debrief (*5 minutes*)

The purpose of this lesson is for students to interpret equations and inequalities expressed using function notation. Students relate them to points $(x, f(x))$ and use the points to sketch a possible graph of the function.

Choose whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion.

- "If $f(7) = 12$, what is true about the graph of the function f?" (The point $(7, 12)$ is a point on the graph.)
- "If $g(5) > g(9)$, what is true about the graph of the function g?" (The point with horizontal value 5 is higher than the point with horizontal value 9.)
- "If $h(8) - h(3) = 11$, what is true about the graph of the function h?" (There is an increase of 11 units between an input of 8 and input of 3.)
- "If $f(x) = y$, what is true about the graph of the function f?" (The point (x, y) is on the graph.)

PLANNING NOTES

Student Lesson Summary and Glossary

What does a statement like $p(3) = 12$ mean?

On its own, $p(3) = 12$ tells us that when p takes 3 as its input, its output is 12. The point $(3, 12)$ is on the graph of the function p.

If we know what quantities the input and output represent, we can learn more about the situation that the function represents.

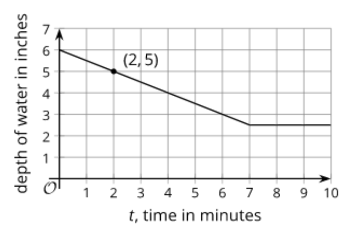
- If function p gives the perimeter of a square whose side length is x and both measurements are in inches, then we can interpret $p(3) = 12$ to mean "a square whose side length is 3 inches has a perimeter of 12 inches."
- We can also interpret statements like $p(x) = 32$ to mean "a square with side length x has a perimeter of 32 inches," which then allows us to reason that x must be 8 inches and to write $p(8) = 32$.
- If function p gives the number of blog subscribers, in thousands, x months after a blogger started publishing online, then $p(3) = 12$ means "3 months after a blogger started publishing online, the blog has 12,000 subscribers."

It is important to pay attention to the units of measurement when analyzing a function. Otherwise, we might mistake what is happening in the situation. If we miss that $p(x)$ is measured in thousands, we might misinterpret $p(x) = 36$ to mean "there are 36 blog subscribers after x months," while it actually means "there are 36,000 subscribers after x months."

A graph of a function can likewise help us interpret statements in function notation.

Function f gives the depth, in inches, of water in a tub as a function of time, t, in minutes, since the tub started being drained.

Here is a graph of f.



Each point on the graph has the coordinates $(t, f(t))$, where the first value is the input of the function and the second value is the output.

- $f(2)$ represents the depth of water 2 minutes after the tub started being drained. The graph passes through $(2, 5)$, so the depth of water is 5 inches when $t = 2$. The equation $f(2) = 5$ captures this information.
- $f(0)$ gives the depth of the water when the draining began, when $t = 0$. The graph shows the depth of water to be 6 inches at that time, so we can write $f(0) = 6$.

- $f(t) = 3$ tells us that t minutes after the tub started draining, the depth of the water was 3 inches. The graph shows that this happens when t is 6.
- $f(7) = f(10)$ tells us that at 7 minutes and at 10 minutes after the tub started draining, the depth of the water was the same. The graph shows that at both times the depth is 2.5 inches. This can be written as $f(7) = 2.5$ and $f(10) = 2.5$.
- $f(4) - f(2) = -1$ tells us that the depth of the water 4 minutes after the tub started draining is one inch lower than the depth of the water 2 minutes after the tub started draining. The graph shows that $f(4) = 4$ and $f(2) = 5$ so $f(4) - f(2) = 4 - 5 = -1$.

Cool-down: Visitors in a Museum (*5 minutes*)

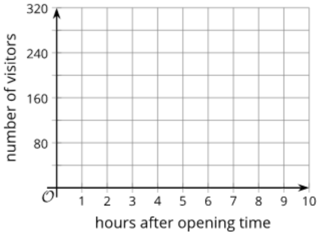
Addressing: NC.M1.A-REI.10; NC.M1.F-IF.2; NC.M1.F-IF.4

Cool-down Guidance: More Chances
Students will have more opportunities to understand the mathematical ideas in this cool-down, so there is no need to slow down or add additional work to the next lessons. Instead, use the results of this cool-down to provide guidance for what to look for and emphasize over the next several lessons to support students in advancing their current understanding.

Cool-down

An art museum opens at 9 a.m. and closes at 5 p.m. The function V gives the number of visitors in a museum h hours after it opens.

1. Explain what this statement tells us about the situation: $V(1.25) = 28$.
2. Use function notation to represent each statement:
   a. At 1 p.m., there were 257 visitors in the museum.
   b. At the time of closing, there were no visitors in the museum.
3. Use the previous statements about the visitors in the museum to sketch a graph that could represent the function.



Student Reflection:

How did you help someone or someone help you in understanding the material today? How did that make you feel about your mathematical ability?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

How did the student work that you selected impact the direction of the discussion? What student work might you pick next time if you taught the lesson again?

Practice Problems

1. Function f gives the temperature, in degrees Celsius, t hours after midnight.

   Choose the equation that represents the statement: "At 1:30 p.m., the temperature was 20 degrees Celsius."

   a. $f(1{:}30) = 20$
   b. $f(1.5) = 20$
   c. $f(13{:}30) = 20$
   d. $f(13.5) = 20$

2. Tyler filled up their bathtub, took a bath, and then drained the tub. The function B gives the depth of the water, in inches, t minutes after they began to fill the bathtub.

   Explain the meaning of each statement in this situation.

   a. $B(0) = 0$

   b. $B(1) < B(7)$

   c. $B(9) = 11$

   d. $B(10) = B(22)$

   e. $B(40) - B(20) = -13$

3. Function f gives the temperature, in degrees Celsius, t hours after midnight.

   Use function notation to write an equation or expression for each statement.

   a. The temperature at 12 p.m.

   b. The temperature was the same at 9 a.m. and at 4 p.m.

   c. It was warmer at 9 a.m. than at 6 a.m.

   d. Some time after midnight, the temperature was 24 degrees Celsius.

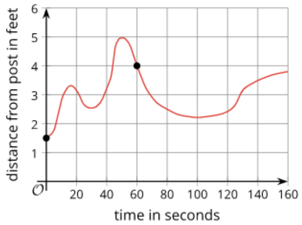
4. Select **all** points that are on the graph of f if we know that $f(2) = -4$ and $f(5) = 3.4$.

   a. $(-4, 2)$
   b. $(2, -4)$
   c. $(3.4, 5)$
   d. $(5, 3.4)$
   e. $(2, 5)$

5. Write three statements that are true about this situation. Use function notation.

   Function f gives the distance of a dog from a post, in feet, as a function of time, t, in seconds, since its owner left.

   Use the $=$ sign in at least one statement and the $<$ sign in another statement.



6. The function $A(x)$, where x is a letter, assigns a value based on the number of the letter in the order of the alphabet. For example, $A(A) = 1$, $A(B) = 2$, $A(C) = 3$, etc. Diego and Andre are trying to figure out whose name as a higher value based on the function $A(x)$ by figuring out the value of each letter in their names and adding them. Whose name has a higher sum based on the $A(x)$ function, Diego or Andre?

   (From Unit 5, Lesson 2)

7. A restaurant owner wants to see if there is a relationship between the amount of sugar in some food items on her menu and how popular the items are.

   She creates a scatter plot to show the relationship between amount of sugar in menu items and the number of orders for those items. The correlation coefficient for the relationship between the two variables is 0.58.

   a. Describe the relationship between the two variables.

   b. Does either of the variables cause the other to change? Explain your reasoning.

   (From Unit 4)

8. Elena writes the equation $6x + 2y = 12$. Write a new equation that has:

   a. exactly one solution in common with Elena's equation

   b. no solutions in common with Elena's equation

   c. infinitely many solutions in common with Elena's equation

   (From Unit 3)

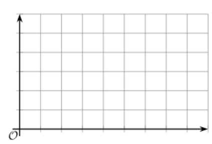
9. Solve the equation: $\frac{2}{3}(12x - 15) + 8 = \frac{1}{4}(8x + 12)$

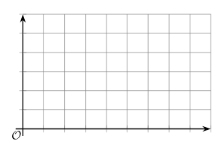
   (From Unit 2)

10. Noah is baking a birthday cake for his dad's 50th birthday party. When he puts the cake in the oven at 2:00, the cake is 0.5 inches high in the pan. Noah checks the cake after 20 minutes to see how much it has risen, it is 1 inch high in the pan. Finally, Noah takes the cake out after another 10 minutes, and it is 1.25 inches high in the pan. If the cake rises at a constant rate as it bakes:

    a. Sketch a graph of the cake's height as a function of the time it baked.

    b. Sketch a graph of the time the cake baked as a function of its height.





    (Addressing NC.8.F.5)

Lesson 4: Using Function Notation to Describe Rules (Part One)

PREPARATION

Lesson Goals	Learning Targets
• Create tables and graphs to represent a function given statements in function notation. • Interpret rules of functions that are expressed using function notation. • Use function notation to write equations that represent rules of functions.	• I can make sense of rules of functions when they are written in function notation and create tables and graphs to represent the functions. • I can write equations that represent the rules of functions.

Lesson Narrative

In earlier lessons, students interpreted and wrote statements in function notation to represent specific input-output pairs of a function (such as $p(2.5) = 18$) or relationships between specific pairs (such as $W(10) = W(12)$).

In this lesson, students learn that function notation can also be used to describe the rule of a function: how a function behaves generally, for any input value. For instance, they see that if the output of a function f can be found by multiplying the input by 3 and then subtracting 10 from the result, we can write $f(x) = 3x - 10$ to represent this rule. We can also use this rule (either the verbal description or the equation) to find the output for any input. In some cases, the rule can also be used to find the input when we know the output.

Students continue to decontextualize given situations into symbolic representations and to contextualize the latter in order to solve problems (MP2). To connect different representations of functions defined by rules, they look for and make use of structure (MP7).

What strategies or representations do you anticipate students might use in this lesson?

Focus and Coherence

Building On	Addressing
NC.8.F.1: Understand that a function is a rule that assigns to each input exactly one output. • Recognize functions when graphed as the set of ordered pairs consisting of an input and exactly one corresponding output.	**NC.M1.A-REI.10:** Understand that the graph of a two variable equation represents the set of all solutions to the equation. **NC.M1.F-BF.1a:** Write a function that describes a relationship between two quantities. **a.** Build linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two ordered pairs (include reading these from a table). (*continued*)

• Recognize functions given a table of values or a set of ordered pairs.	**NC.M1.F-IF.1:** Build an understanding that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range by recognizing that: • if f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. • the graph of f is the graph of the equation $y = f(x)$. **NC.M1.F-IF.2:** Use function notation to evaluate linear, quadratic, and exponential functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **NC.M1.F-IF.7:** Analyze linear, exponential, and quadratic functions by generating different representations, by hand in simple cases and using technology for more complicated cases, to show key features, including: domain and range; rate of change; intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and end behavior.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*10 minutes*)
- **Activity 2** (*15 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U5.L4 Cool-down (print 1 copy per student)

LESSON

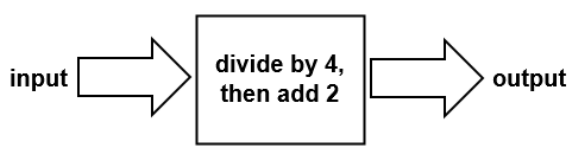
Bridge (*Optional, 5 minutes*)

Building On: NC.8.F.1

The purpose of this bridge is for students to use a rule, given in words, to generate output values for a given set of input values. Later in the lesson, students will be using function rules given in symbolic form to generate input-output pairs. This task is aligned to question 2 in Check Your Readiness.

Student Task Statement

Given the function rule:[1]



Complete the table for the following input values:

Input	0	2	4	6	8	10
Output						

[1] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

DO THE MATH	PLANNING NOTES

Warm-up: Two Functions (*5 minutes*)

Instructional Routine: Notice and Wonder	
Addressing: NC.M1.F-IF.2	**Building Towards:** NC.M1.F-BF.1

This warm-up familiarizes students with a new way of using function notation and gives them a preview of the work in this lesson.

The prompt also gives students opportunities to see and make use of structure (MP7). The specific structure they might notice is how the values in the $f(x)$ and the $g(x)$ columns in each table correspond to the expression describing each function.

When students articulate what they notice and wonder, they have an opportunity to attend to precision in the language they use to describe what they see (MP6). They might first use less formal or imprecise language, and then restate their observation with more precise language in order to communicate more clearly.

Step 1

- Display the tables for all to see. Use the *Notice and Wonder* routine and ask students, "What do you notice? What do you wonder?"
- Give students a minute to think of things they notice and things they wonder and then share them with a partner.

NOTICE AND WONDER	***What Is This Routine?*** Students are shown some media or a mathematical representation. The prompt to students is "What do you notice? What do you wonder?" Students are given a few minutes to think of things they notice and things they wonder, and share them with a partner. Then, the teacher asks several students to share things they noticed and things they wondered; these are recorded by the teacher for all to see. Sometimes the teacher steers the conversation to wondering about something mathematical that the class is about to focus on. ***Why This Routine?*** The purpose of the *Notice and Wonder* routine is to make a mathematical task accessible to all students with these two approachable questions. By thinking about them and responding, students gain entry into the context and might get their curiosity piqued. Taking steps to become familiar with a context and the mathematics that might be involved is making sense of problems (MP1).

Student Task Statement

What do you notice? What do you wonder?

x	$f(x) = 10 - 2x$
1	8
1.5	7
5	0
-2	14

x	$g(x) = 3x^2$
-2	12
0	0
1	3
3	27

Step 2

- Facilitate a whole-class discussion by asking students to share the things they noticed and wondered.
- Record and display their responses for all to see. If possible, record the relevant reasoning on or near the tables.
- After all responses have been recorded without commentary or editing, ask students, "Is there anything on this list that you are wondering about?" Encourage students to respectfully disagree, ask for clarification, or point out contradicting information.

DO THE MATH

PLANNING NOTES

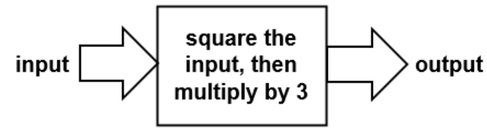
Activity 1: Four Functions (*10 minutes*)

Instructional Routine: Discussion Supports (MLR8) - Responsive Strategy	
Building On: NC.8.F.1	**Addressing:** NC.M1.F-IF.1

In this activity, students are introduced to the idea that some functions can be defined by a rule, and the rule can be described in words or with expressions and equations. Students examine some simple rules and make connections between their verbal and algebraic representations. Doing so prompts them to look for and make use of structure (MP7).

The algebraic statements are written in function notation, so the work also reinforces students' understanding of the notation and expands their capacity to use it to describe functions.

Step 1



- Display an image of a "function machine" with "square the input then multiply by 3" as the rule.
- Tell students that a function takes any input, squares it, and then multiplies by 3 to generate the output. Ask students to
  - Find the output when the inputs are -2, 0, 1, 3, and x.
  - Display this table and complete it with the values provided by students.

x	$g(x)$
-2	
0	
1	
3	
x	

  - If not mentioned by students, point out that these equations describe the same function as that shown by the second table in the warm-up.
- Explain to students that some functions have a specific rule for getting its output. The rule can be described in words (like "square the input and multiply by 3") or with expressions (such as $3x^2$). Tell students that they'll now look at some rules expressed in both ways.

RESPONSIVE STRATEGY

Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity. Give students a subset of the expressions and descriptions to start with and hold a brief small-group or whole-class discussion once students have completed the first match. Invite 1–2 students to share their strategies for finding a match before introducing the remaining expressions and descriptions.

Supports accessibility for: Conceptual processing; Organization

Student Task Statement

Here are descriptions and equations that represent four functions.

Descriptions	Equations
a. To get the output, subtract 7 from the input, then divide the result by 3.	$f(x) = 3x - 7$
b. To get the output, subtract 7 from the input, then multiply the result by 3.	$g(x) = 3(x - 7)$
c. To get the output, multiply the input by 3, then subtract 7 from the result.	$h(x) = \frac{x}{3} - 7$
d. To get the output, divide the input by 3, then subtract 7 from the result.	$k(x) = \frac{x-7}{3}$

1. Match each equation with a verbal description that represents the same function. Record your results.
2. For one of the functions, when the input is 6, the output is -3. Which is that function: f, g, h, or k? Explain how you know.
3. Which function value—$f(x)$, $g(x)$, $h(x)$, or $k(x)$—is the greatest when the input is 0? What about when the input is 10?

Are You Ready For More?

Mai says $f(x)$ is always greater than $g(x)$ for each value of x. Is this true? Explain how you know.

Step 2

- Give students 2 minutes of individual think time to begin answering the questions in the activity.
- Have students form pairs and give them 2 minutes to share their responses and discuss any discrepancies.

Step 3

- Invite students to briefly share how they matched the equations and verbal descriptions in the first question. Discuss questions such as:
  - "The expressions for functions f and g both involve multiplying by 3 and subtracting 7. How are they different?" (The order in which the operations happen is different. Function f first multiplies the input by 3, and then 7 is subtracted from the result. Function g first subtracts 7, then multiplies the result by 3.)
- "The expressions for h and k both involve subtracting 7 and dividing by 3. How did you decide which one corresponds to description 'a' and which one corresponds to 'd'?" (By looking at what is done to x first. In h, x is divided by 3 before 7 is subtracted, so it must correspond to "d.")
- Next, ask students how they determined which function has $(6, -3)$ as an input-output pair and which function has the greatest output when x is 0 and when x is 10. Highlight explanations that mention evaluating each function at those input values and seeing which one generates -3 for the output or gives the greatest output.
- The functions in this activity are given without a context. Tell students that they will now look at rules that describe relationships between quantities in situations.

RESPONSIVE STRATEGY

Use this routine to support whole-class discussion as students explain how they matched the descriptions to the equations. Display the following sentence frames for all to see: "The equation ____ matches ____ because . . ." and "I noticed ____, so . . . ." Encourage students to challenge each other when they disagree. If necessary, revoice student ideas to demonstrate using mathematical language such as input and output. This will help students use the structure of equations to make connections between equations and descriptions of functions.

Discussion Supports (MLR8)

DO THE MATH	PLANNING NOTES

Activity 2: Rules for Area and Perimeter (*15 minutes*)

Instructional Routine: Collect and Display (MLR2)	
Addressing: NC.M1.A-REI.10; NC.M1.F-BF1.a; NC.M1.F-IF.2; NC.M1.F-IF.7	

Previously, students interpreted rules of functions only in terms of the operations performed on the input to lead to the output. In this activity, students analyze functions that relate two quantities in a situation and work to define the relationship between the quantities with a rule. They do so by creating a table of values and generalizing the process of finding one quantity given the other. Students also plot the values in each table to see the graphical representation of the functions.

The mathematical reasoning here is not new. Students have done similar work earlier in the course when investigating expressions and equations. What is new is seeing these relationships as functions and using function notation to describe them.

Step 1

- Ask students to arrange themselves into pairs or use visibly random grouping.
- Give students a few minutes of quiet time to work on question 1 and then a moment to discuss their responses with their partner.

Student Task Statement

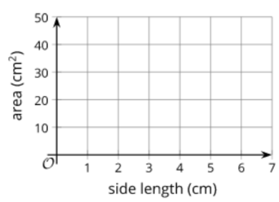
1. A square that has a side length of 9 cm has an area of 81 cm^2. The relationship between the side length and the area of the square is a function.

    a. Complete the table with the area for each given side length.

    Then, write a rule for a function A that gives the area of the square in cm^2 when the side length is s cm. Use function notation.

    b. What does $A(2)$ represent in this situation? What is its value?

    c. On the coordinate plane, sketch a graph of this function.



Side length (cm)	Area (cm^2)
1	
2	
4	
6	
s	

Step 2

- Facilitate a brief discussion. Invite students to share their rule for the area function. Some students may have written $A = s^2$, while others may have written $A(s) = s^2$. Ask students who wrote each way to explain their reasoning. Highlight explanations that point out that A is the name of the function and that function notation requires specifying the input, which is s.
- Clarify that in the past, we may have used a variable like A to represent the area, but in this case, A is used to name a function to help us talk about its input and output. If we wish to also use a variable to represent the output of this function (instead of using function notation), it would be helpful to use a different letter.

Step 3

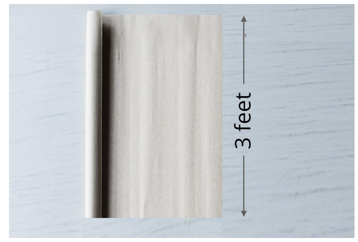
- Give students a few minutes of quiet time to work on question 2, and then time to discuss their responses with their partner.

Monitoring Tip: Students are likely to graph the functions by plotting the values in the tables and then connecting the points with a curve or line. As students work on the second set of questions about a perimeter function, which is linear, look for those who relate $P(l) = 2l + 6$ to a linear equation, namely $y = 2x + 6$, and then graph a line with a vertical intercept of $(0, 6)$ and a slope of 2. Let them know that they may be asked to share later.

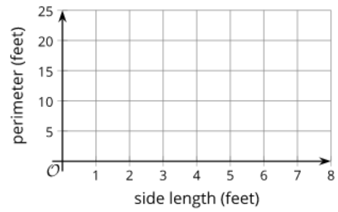
***Advancing Student Thinking*:** If students struggle to graph the functions, suggest that they use the coordinate pairs in the tables to help them.

Student Task Statement

2. A roll of paper that is 3 feet wide can be cut to any length.
   a. If we cut a length of 2.5 feet, what is the perimeter of the paper?
   b. Complete the table with the perimeter for each given side length. Then, write a rule for a function P that gives the perimeter of the paper in feet when the side length in feet is l. Use function notation.



Side length (feet)	Perimeter (feet)
1	
2	
6.3	
11	
ℓ	



   c. What does $P(11)$ represent in this situation? What is its value?
   d. On the coordinate plane, sketch a graph of this function.

Step 4

- Use the *Collect and Display* routine by asking selected students to share the rule they wrote for the perimeter function and how they determined the rule. Students may have written expressions of different forms for $P(l) : l + l + 3 + 3;\ 2l + 2(3);\ 2(l + 3);\ 6 + 2l$.
- Record and display variations both in the way students expressed the rule algebraically and in the words students use to describe their function. Ask students to explain how they know these expressions are equivalent and define the same function. Continue to annotate the expressions and scribe student language for all to see.
- Next, select previously identified students to share how they sketched the graph of the function. If no students made a connection between the slope and vertical intercept of the graph of P to the parameters in their equation, ask them about it. For example, display the graph of P and ask students to use it to write an equation for the line.

COLLECT AND DISPLAY	***What Is This Routine?*** The teacher captures students' oral words and phrases into a stable, collective reference in order to stabilize the fleeting language that students use during partner, small-group, or whole-class activities. The teacher listens for, and scribes, the student output using written words, diagrams, and pictures; this collected output can be organized, revoiced, or explicitly connected to other language in a display for all students to use over the course of a lesson or unit. ***Why This Routine?*** *Collect and Display* (MLR2) provides feedback for students in a way that increases accessibility while simultaneously supporting meta-awareness of language. The routine mirrors student language back to the whole class to enable students' own output to be used as a reference in developing their mathematical language over time.

DO THE MATH	PLANNING NOTES

Lesson Debrief (*5 minutes*)

The purpose of the lesson is for students to use a function rule to create tables and graphs of the function. Students also write function rules for a context.

Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion.

Display for all to see the equations $f(x) = 5x + 3$ and $g(x) = 10x - 4$. Ask students:

- "How would you describe to a classmate who is absent today what each equation means? What would you say to help them make sense of these?" (Each equation gives the rule of a function. The rule for f says that, to get the output, we multiply the input by 5 and add 3. The rule for g says that the output is 10 times the input, minus 4.)
- "How do the rules help us find the value of $f(10)$ or $g(10)$?" (If we substitute 10 for x in each equation and evaluate the expression, we would have the value of f or g at $x = 10$, which are 53 and 96, respectively.)
- "Is it possible to graph a function described this way? How?" (We could create a table of values and find the coordinate pairs at different x-values. Or, if a rule is expressed as a linear equation, we could use it to identify the slope and vertical intercept of the graph.)

PLANNING NOTES

Student Lesson Summary and Glossary

Some functions are defined by rules that specify how to compute the output from the input. These rules can be verbal descriptions or expressions and equations. For example:

Rules in words:

- To get the output of function f, add 2 to the input, then multiply the result by 5.
- To get the output of function m, multiply the input by $\frac{1}{2}$ and subtract the result from 3.

Rules in function notation:

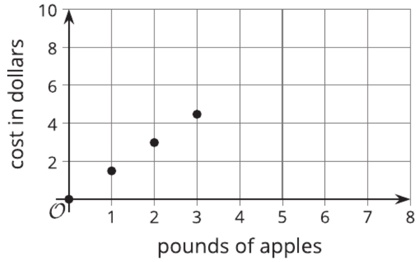
- $f(x) = (x+2) \cdot 5$ or $5(x+2)$
- $m(x) = 3 - \frac{1}{2}x$

Some functions that relate two quantities in a situation can also be defined by rules and can therefore be expressed algebraically, using function notation. Suppose function c gives the cost of buying n pounds of apples at $\$1.49$ per pound. We can write the rule $c(n) = 1.49n$ to define function c.

To see how the cost changes when n changes, we can create a table of values.

Pounds of apples, n	Cost in dollars, $c(n)$
0	0
1	1.49
2	2.98
3	4.47
n	$1.49n$

Plotting the pairs of values in the table gives us a graphical representation of c.



Cool-down: Perimeter of a Square (*5 minutes*)

Addressing: NC.M1.A-REI.10; NC.M1.F-BF.1.a; NC.M1.F-IF.2

Cool-down Guidance: Points to Emphasize
If students continue to struggle with function notation, look carefully at student work and use the activity synthesis in Activity 1 of Lesson 6 to clarify misconceptions from student work. If there is time, consider having students revise their cool-downs or use Lesson 4 practice problems 3 or 4 as an additional assessment opportunity.

Cool-down

1. Complete the table with the perimeter of a square for each given side length.
2. Write a rule for a function P that gives the perimeter of a square in inches when the side length is x inches.
3. What is the value of $P(9.1)$? What does it tell us about the side length and perimeter of the square?

Side length (inches)	Perimeter (inches)
0.5	
7	
20	

Student Reflection:

- What is one concept you struggled with today?
- What do you need to help you understand better?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

Think about which students haven't shared their strategies in class lately. Were there missed opportunities to highlight their thinking during recent lessons? How can you take advantage of those opportunities when they arise?

Practice Problems

1. Match each equation with a description of the function it represents.
   a. $f(x) = 2x + 4$
   b. $g(x) = 2(x + 4)$
   c. $h(x) = 4x + 2$
   d. $k(x) = 4(x + 2)$

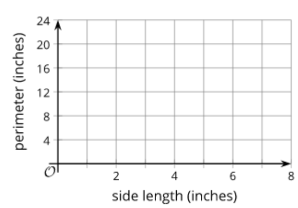
   1. To get the output, add 4 to the input, then multiply the result by 2.
   2. To get the output, add 2 to the input, then multiply the result by 4.
   3. To get the output, multiply the input by 2, then add 4 to the result.
   4. To get the output, multiply the input by 4, then add 2 to the result.

2. Function P represents the perimeter, in inches, of a square with side length x inches.
   a. Complete the table.

x	0	1	2	3	4	5	6
$P(x)$							

   b. Write an equation to represent function P.

   c. Sketch a graph of function P.



3. Functions f and A are defined by these equations.

$$f(x) = 80 - 15x$$

$$A(x) = 25 + 10x$$

   a. Which function has a greater value when x is 2.5?

   b. Which function has a greater value when x is -2.5?

4. An equilateral triangle has three sides of equal length. Function P gives the perimeter of an equilateral triangle of side length s.

   a. Find $P(2)$
   b. Find $P(10)$
   c. Find $P(s)$

5. Function C gives the cost, in dollars, of buying n apples. Which statement best represents the meaning of $C(10) = 9$?

   a. The cost of buying 9 apples
   b. The cost of 9 apples is $10.
   c. The cost of 10 apples
   d. Ten apples cost $9.

   (From Unit 5, Lesson 2)

6. Imagine a situation where a person is using a garden hose to fill a child's pool. Think of two quantities that are related in this situation and that can be seen as a function.

   a. Define the function using a statement in the form "___ is a function of ____." Be sure to consider the units of measurement.

   b. Sketch a possible graph of the function. Be sure to label the axes.

      Then, identify the coordinates of one point on the graph and explain its meaning.

   (From Unit 5, Lesson 1)

7. Diego is baking cookies for a fundraiser. He opens a 5-pound bag of flour and uses 1.5 pounds of flour to bake the cookies.

   Which equation or inequality represents f, the amount of flour left in the bag, after Diego bakes the cookies?

   a. $f = 1.5$
   b. $f < 1.5$
   c. $f = 3.5$
   d. $f > 3.5$

   (From Unit 2)

8. The data set represents the number of cars in a town given a speeding ticket each day for 10 days.

   2 4 5 5 7 7 8 8 8 12

   a. What is the median? Interpret this value in the situation.

   b. What is the IQR?

   (From Unit 1)

9. Mai took a survey of students in her class to find out how many hours they spend reading each week. Here are some summary statistics for the data that Mai gathered:

   - mean: 8.5 hours
   - standard deviation: 5.3 hours
   - median: 7 hours
   - Q1: 5 hours
   - Q3: 11 hours

   a. Give an example of an outlier, and explain your reasoning.

   b. Are there any outliers below the median? Explain your reasoning.

   (From Unit 1)

Lesson 5: Using Function Notation to Describe Rules (Part Two)

PREPARATION

Lesson Goals	Learning Targets
• Evaluate functions and solve equations given in function notation, either by graphing or by reasoning algebraically. • Understand linear function as a function whose output changes at a constant rate and whose graph is a line. • Use technology to graph and evaluate functions given in function notation.	• I know different ways to find the value of a function and to solve equations written in function notation. • I know what makes a function a linear function. • I can use technology to graph a function given in function notation and use the graph to find the values of the function.

Lesson Narrative

In an earlier lesson, students learned that some functions can be defined with a rule and the rule can be expressed using function notation. In this lesson, students use rules of functions to find the output when the input is given (evaluating functions) and to find the input when the output is known (solving equations that define functions). They also interpret rules of functions in terms of a situation. Along the way, they practice reasoning quantitatively and abstractly (MP2).

The term **linear function** is introduced here. In middle school, students learned that a relationship between two quantities is linear if one quantity changes at a constant rate relative to the other. Students see that a linear function can be understood in similar terms: a function is linear if the output changes by a constant rate relative to its input.

This lesson also includes an activity that is designed to enable students to use technology to graph and evaluate functions expressed in function notation. This skill can help to develop students' understanding of functions and ability to solve problems in this unit and in future units.

What aspects of your professional learning will you think about during this lesson?

Focus and Coherence

Building On	Addressing
NC.8.F.4: Analyze functions that model linear relationships. • Understand that a linear relationship can be generalized by $y = mx + b$. • Write an equation in slope-intercept form to model a linear relationship by determining the rate of change and the initial value, given at least two (x, y) values or a graph. • Construct a graph of a linear relationship given an equation in slope-intercept form. • Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of the slope and y-intercept of its graph or a table of values.	**NC.M1.A-REI.3:** Solve linear equations and inequalities in one variable. **NC.M1.A-REI.10:** Understand that the graph of a two variable equation represents the set of all solutions to the equation. **NC.M1.F-IF.2:** Use function notation to evaluate linear, quadratic, and exponential functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **NC.M1.F-IF.7:** Analyze linear, exponential, and quadratic functions by generating different representations, by hand in simple cases and using technology for more complicated cases, to show key features, including: domain and range; rate of change; intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and end behavior.

Agenda, Materials, and Preparation

- **Warm-up** (*5 minutes*)
- **Activity 1** (*20 minutes*)
  - Blank visual displays for each student pair (possible visual display options: poster board, chart paper, Google Slides, Jamboard)
- **Activity 2** (*10 minutes*)
  - Graphing technology is required: Acquire devices that can access Desmos (recommended) or other graphing technology. It is ideal if each student has their own device.
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U5.L5 Cool-down (print 1 copy per student)

LESSON

Warm-up: Make It True (*5 minutes*)

Building On: NC.8.F.4	**Addressing:** NC.M1.A-REI.3

This warm-up refreshes the idea of evaluating expressions and solving equations, preparing students for the main work of the lesson. It reminds students that to solve a variable equation is to find one or more values for the variable that would make the equation true. In subsequent activities, students will work with equations involving function notation to find unknown input or output values.

Step 1

- Ask students to arrange themselves into pairs or use visibly random grouping.
- Provide students with 2 minutes of quiet think time and then time to discuss with their partner.

Student Task Statement

Consider the equation $q = 4 + 0.8p$. Explain or show your reasoning.

1. What value of q would make the equation true when:
   a. p is 7?
   b. p is 100?
2. What value of p would make the equation true when:
   a. q is 12?
   b. q is 60?

Step 2

- Invite students to share their responses and strategies for answering the questions.
- To find the values of p in the second set of questions, some students may have guessed and checked, but many students should have recognized that they could solve the equations for p (either before or after substituting the value of q). If no students mentioned solving the equations, bring it to their attention.
- Remind students that to solve $12 = 4 + 0.8p$ is to find the value of p that makes the equation true.

DO THE MATH	PLANNING NOTES

Activity 1: Data Plans (*20 minutes*)

Instructional Routine: Compare and Connect (MLR7)
Addressing: NC.M1.A-REI.10; NC.M1.F-IF.2; NC.M1.F-IF.7

In this activity, students work with rules of functions that arise from situations and continue to make connections between different representations of functions. They use rules to evaluate functions and interpret the input and output values in context. For instance, they see that $B(1)$ represents the cost of using 1 gigabyte of data beyond the monthly allowance of data plan B, and find its value by computing $10(1) + 25$.

Students also use rules of functions to solve for an unknown input value. For example, given the rule $B(x) = 10x + 25$ and the statement $B(x) = 50$, they work to find a value of x that makes $B(x) = 50$ true.
Note that the discussion avoids the term "y-value" when referring to the height of a point on a graph. Because the problems in this activity (and many other problems involving function notation) do not use the variable y, it is more appropriate to refer to the "vertical value."

Step 1

- Keep students in pairs.
- As a class, read the opening paragraphs and the first question in the activity statement.
- Ask students, "What do $A(1)$ and $B(1)$ represent in this situation? What does the 1 in each expression mean?" Give students a moment of quiet think time and then time to discuss their thinking with their partner.
- Invite students to share their interpretations, in particular the meaning of "allowance." Most data plans include a basic amount of data at a fixed cost but then start charging an additional fee if usage goes beyond this allowance. Before students begin the activity, make sure they see that an input value of 1 represents usage of 1 gigabyte of data beyond the monthly allowance.

Step 2

- Provide students time to complete the activity with their partner.

> **Monitoring Tip:** Look for the different ways students find the value of x such that $B(x) = 50$. This may include:
> - guessing and checking
> - analyzing the graph of $y = 10x + 25$ and identifying the x-value when y is 50
> - reasoning that a budget of $50 means that only $25 is available for extra data, and at $10 per gigabyte, it means 2.5 gigabytes of data
> - solving $10x + 25 = 50$ algebraically
>
> Let them know that they may be asked to share later. Include at least one student who does not typically volunteer.

- Invite students to create a visual display that shows their strategy for the last question.

> ***Advancing Student Thinking***: Students may question if A is a function at all, because unlike B or other function rules they have seen so far, $A(x)$ is defined with a constant instead of an expression containing the dependent variable. Or they may wonder why $A(x)$ has the same value no matter what the input value is. Ask students to recall the definition of function and to consider whether each input value gives only one output value. Because it does, even though it is always the same output value, A is still a function.

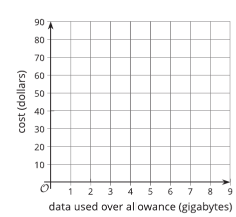
Student Task Statement

A college student is choosing between two data plans for her new cell phone. Both plans include an allowance of 2 gigabytes of data per month. The monthly cost of each option can be seen as a function and represented with an equation:

- Option A: $A(x) = 60$
- Option B: $B(x) = 10x + 25$

In each function, the input, x, represents the gigabytes of data used *over* the monthly allowance.

1. The student decides to find the values of $A(1)$ and $B(1)$ and compare them. What are those values?
2. After looking at some of her past phone bills, she decided to compare $A(7.5)$ and $B(7.5)$. What are those values?
3. Describe each data plan in words.
4. Graph each function on the same coordinate plane. Then, explain which plan you think she should choose.
5. The student only budgeted $50 a month for her cell phone. She thought, "I wonder how many gigabytes of data I would have for $50 if I go with Option B?" and wrote $B(x) = 50$. What is the answer to her question? Explain or show how you know.



Are You Ready For More?

Describe a different data plan that, for any amount of data used, would cost no more than one of the given plans and no less than the other given plan. Explain or show how you know this data plan would meet these requirements.

Step 3

- Select students to share their interpretations of the two data plans. Make sure students see that:
  - The equation $A(x) = 60$ tells us that, regardless of the extra gigabytes of data used, x, the cost, $A(x)$, is always 60.
  - The $10x$ in the rule of $B(x)$ tells us that each extra gigabyte of data used costs $10, and that there is a $25 fixed fee.
- Explain to students that the two functions here are linear functions because the output of each function changes at a constant rate relative to the input. Option B involves a rate of change of $10 per gigabyte of data over the monthly allowance, while option A has a rate of change of $0 per gigabyte over the allowance.
- Ask students how they went about graphing the functions. Students are likely to have plotted some input-output pairs of each function. If no students mention identifying the slope and vertical intercept of each graph, ask them about it.

RESPONSIVE STRATEGIES

Use color and annotations to illustrate student thinking. As students share their reasoning about the two data plans and the graphs that represent the linear functions of each, scribe their thinking on a visible display.

Supports accessibility for: Visual-spatial processing; Conceptual processing

Use the *Compare and Connect* routine to focus the discussion on students' response to the last question and how they found out the gigabytes of data that could be bought with $50 under option B.

- Display the strategies to the last question.
- Allow students time to quietly circulate and analyze the strategies in at least two other displays in the room.
- Give students quiet think time to consider how the strategies are alike and how they are different.
- Ask students to find a partner to discuss what they noticed. This will help students make connections between the strategies used to solve for an unknown input value.

- Select previously identified students to share their strategies, in the order listed in the Monitoring Tip. If no one mentions using a graph or solving $10x + 25 = 50$, bring these up.
- Explain the following points to help students connect some key ideas:
  - We can graph functions like $A(x) = 60$ and $B(x) = 10x + 25$ without plotting individual coordinate pairs.
    - $A(x)$ is the output of function A and is represented by vertical values on a coordinate plane. The vertical values are typically labeled with the variable y, so we can write $y = A(x)$ and graph $y = 60$ to represent function A.
    - Likewise, $B(x)$ is the output of function B and is represented by vertical values on a plane. We can write $y = B(x)$ and graph $y = 10x + 25$ to represent function B.
  - To solve equations like $B(x) = 50$ means to find one or more values of x that make the equation true. We can do this, among other ways, by using the graph of B or by solving an equation algebraically.

- On the graph of B, we can look for one or more values of x that correspond to the vertical value of 50. This might involve some estimating.
- Because $B(x)$ is equal to $10x+25$ and $B(x)$ is also equal to 50, we can write $10x+25=50$ and solve the equation.

- If time permits, invite students to share which option they believe the college student should choose and why.

DO THE MATH	PLANNING NOTES

Activity 2: Function Notation and Graphing Technology (*10 minutes*)

Instructional Routines: Graph It; Discussion Supports (MLR8) - Responsive Strategy
Building Towards: NC.M1.A-REI.10; NC.M1.F-IF.2; NC.M1.F-IF.7

In this *Graph It* activity, students learn to use graphing technology to graph an equation in function notation, evaluate the function at a specific input value, create a table of inputs and outputs, and identify the coordinates of the points along a graph. They also revisit how to set an appropriate graphing window.

GRAPH IT	***What Is This Routine?*** *Graph It* indicates activities where students have an opportunity to use graphing technology to visualize a graph representing one or more functions with known parameters and use the tool to find features like intersection points, intercepts, and maximums or minimums. Additionally, they may use sliders for exploring the effect of changing parameters. ***Why This Routine?*** Using graphs to solve problems can be cognitively demanding for students not yet fluent with creating this type of representation. Through accessing technology to assist with exploring the nature of graphs, the *Graph It* routine provides students a scaffold for interpreting the structure of the coordinate plane and developing fluency in creating graphs on their own.

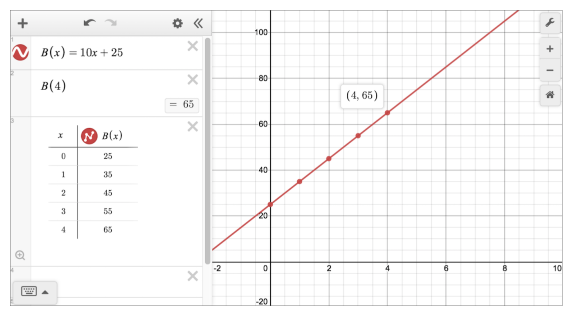
Step 1

- Display the equation $B(x)=10x+25$ for all to see. Ask students:
  - "How would you go about finding the value of $B(1.482)$?" (Substitute 1.482 into the expression $10x+25$ and evaluate, or use the graph of $y=10x+25$ to estimate the y-value when x is 1.482.)
  - "How would you find the value of x that makes $B(x)=103.75$ true?" (Solve $10x+25=103.75$, or use the graph of $y=10x+25$ to estimate the x-value when y is 103.75.)
- Explain that while it's possible to use a graph to find or estimate unknown input or output values, it is hard to be precise when using a hand-drawn or printed graph. We can evaluate the expression or solve the equation algebraically, but computing by hand can get cumbersome (though a calculator can take care of the most laborious part). Let's see how graphing technology can help us!

Step 2

- Use Desmos and display for all to see. Point out the column on the left, which shows blank rows. We can use them to enter expressions and create a list. Demonstrate the following:
  - In the first row of the expression list, type: $B(x) = 10x + 25$. A graph appears, but the line might appear to be squeezed up against the vertical axis.
  - Click the "Graph Settings" button (wrench icon) in the upper right corner to change the graphing window. Experiment with the graphing window until the graph shows more information or seems more useful.
  - Ask students to find $B(4)$ on the graph. Then, demonstrate some ways (other than approximating visually) to find $B(4)$ more precisely:
    - Trace the line. Coordinates appear as we move along the line.
    - Type $B(4)$ in the expression list. A small rectangle appears showing the value.
    - Click the wheel icon at the upper right, above the line $B(x) = 10x + 25$. A table icon will appear next to the function. Click this icon to create a table with x as the input and $B(x)$ as the output. If we enter 4 in a cell for x, the value of $B(x)$ is calculated automatically.

      Here is a screenshot showing these features.



- Ask students to find the value of x when $B(x)$ is 100. Then, demonstrate some ways (other than approximating visually) to solve for x given $B(x) = 100$:
  - Trace the line. Coordinates appear as we move along the line. Stop when the x-coordinate is 100 and see what the y-coordinate is.
  - Type $y = 100$ in the expression list. A horizontal line appears. Click on the intersection of this line and the graph of B.
- Ask students to use one or both of these strategies to complete the activity.

RESPONSIVE STRATEGY

Support effective and efficient use of tools and assistive technologies. To use graphing technology, some students may benefit from a demonstration or access to step-by-step instructions.

Supports accessibility for: Organization; Memory; Attention

Student Task Statement

The function B is defined by the equation $B(x) = 10x + 25$. Use graphing technology to:

1. Find the value of each expression:

   a. $B(6)$ b. $B(2.75)$ c. $B(1.482)$ d. $B(-3.5)$

2. Solve each equation:

   a. $B(x) = 93$ b. $B(x) = 42.1$ c. $B(x) = -78$ d. $B(x) = 116.25$

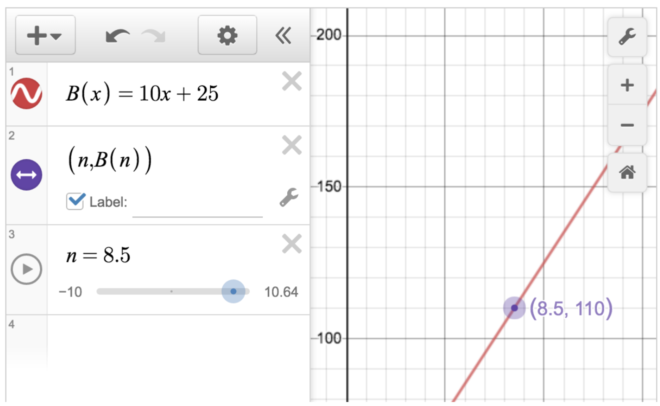
Step 3

- Invite students to share any insights they had while using the graphing tool and techniques to evaluate expressions and solve equations. In what ways might the tool and techniques be handy? When might they be limited?
- Also discuss any issues that students encountered while completing the task—technical or otherwise.
- If desired, consider showing another way to obtain input-output pairs of a function in Desmos.
  - Let's assign a new input variable, say, n, to function B. If we enter $(n, B(n))$ in the expression list, activate a slider for n, and enable the option to label points, the graph will show the coordinate pair for any value of n.

RESPONSIVE STRATEGY

Give students additional time to share their insights in small groups before the whole-class discussion. Invite groups to discuss the advantages and disadvantages of using the graphing tool and to rehearse what they will say when they share with the whole class. Rehearsing provides students with additional opportunities to speak and clarify their thinking, and will improve the quality of explanations shared during the whole-class discussion.

Discussion Supports (MLR8)



DO THE MATH

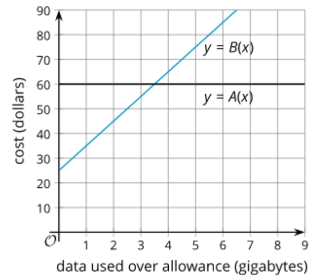
PLANNING NOTES

Lesson Debrief (*5 minutes*)

The purpose of this lesson is for students to analyze a linear function and make connections between different representations. Students interpret input and output values in context, use rules of functions to solve for unknown inputs, and graph linear functions.

Choose whether students should reflect on the first three bulleted items in their workbooks or talk through them with a partner.

Refer back to the activity about data plans. Display the two graphs for all to see.



- Ask students to use the displayed graphs to find:
  - the values of $A(0.8)$ and $B(0.8)$
  - the solutions to $A(x) = 72$ and $B(x) = 72$
- Next, display the two equations for all to see. (Hold off on discussing students' responses.)

  $A(x) = 60$

  $B(x) = 10x + 25$
- Ask students to use the equations to find the same four values as they have just found using the graphs.
- Invite students to compare and contrast the graphical and algebraic approaches for finding unknown inputs and outputs of linear functions. Discuss questions such as:
  - "How easy was it to use the graph of B to find an output value such as $B(0.8)$? What about using the graph to find an input value, such as the x in $B(x) = 72$?" (Both were fairly straightforward but may not have been very precise. Some estimation was necessary. It was very easy to see that there was no solution to $A(x) = 72$.)
  - "How easy was it to use the rule $B(x) = 10x + 25$ to find an output value such as $B(0.8)$? What about finding an input value, such as the x in $B(x) = 72$?" (Both were fairly simple, but if the rule or the given input or output involves numbers that are harder to compute by hand, it might be more complicated.)
- Ask students to identify some ways that technology could help to find unknown input and output values of a function.

PLANNING NOTES

Student Lesson Summary and Glossary

Knowing the rule that defines a function can be very useful. It can help us to:

- Find the output when we know the input.
  - If the rule $f(x) = 5(x + 2)$ defines f, we can find $f(100)$ by evaluating $5(100 + 2)$.
  - If $m(x) = 3 - \frac{1}{2}x$ defines function m, we can find $m(10)$ by evaluating $3 - \frac{1}{2}(10)$.

- Create a table of values.

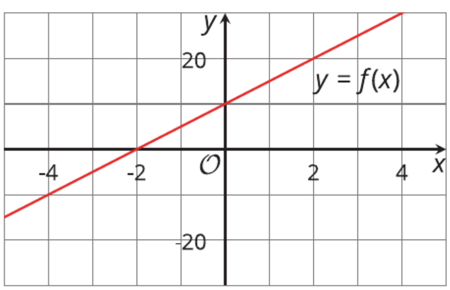
  Here are tables representing functions f and m:

x	$f(x) = 5(x+2)$
-2	0
-1	5
0	10
1	15
2	20
3	25
4	30

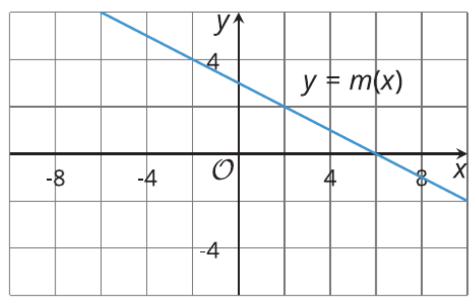
x	$m(x) = 3 - \frac{1}{2}x$
-2	4
-1	$3\frac{1}{2}$
0	3
1	$2\frac{1}{2}$
2	2
3	$1\frac{1}{2}$
4	1

- Graph the function. The horizontal values represent the input, and the vertical values represent the output.

  For function f, the values of $f(x)$ are the vertical values, which are often labeled y, so we can write $y = f(x)$. Because $f(x)$ is defined by the expression $5(x+2)$, we can graph $y = 5(x+2)$.



  For function m, we can write $y = m(x)$ and graph $y = 3 - \frac{1}{2}x$.



- Find the input when we know the output.

  - Suppose the output of function f is 65 at some value of x, or $f(x) = 65$, and we want to find out what that value is. Because $f(x)$ is equal to $5(x+2)$, we can write $5(x+2) = 65$ and solve for x.

$$5(x+2) = 65$$
$$x + 2 = 13$$
$$x = 11$$

Each function here is a **linear function** because the value of the function changes by a constant rate and its graph is a line.

Linear function: A function that has a constant rate of change. Another way to say this is that it grows by equal differences over equal intervals. For example, $f(x) = 4x - 3$ defines a linear function. Any time x increases by 1, $f(x)$ increases by 4.

Cool-down: A Third Option (*5 minutes*)

Addressing: NC.M1.A-REI.10; NC.M1.F-IF.2

Cool-down Guidance: Press Pause
At this point, students need to be able to evaluate and interpret equations given in function notation. If students continue to struggle, select examples of cool-downs from this unit to highlight and clarify misconceptions. Practice problems 1, 3, 4, and 5 from this lesson all provide opportunities for practice and additional formative assessment.

Cool-down

The college student who is looking for a data plan for her cell phone found a third option, which also offers a 2 gigabyte monthly allowance for data.

The monthly cost of this option can be represented by function C, defined by: $C(x) = 30x + 5$, where x is the gigabytes of data used *over* the monthly allowance.

1. Find $C(0.7)$ and explain what it means in this situation.
2. Her budget is still $50 a month. To find out how many gigabytes of data she could use if she chose this plan, she writes $C(x) = 50$ and solves for x. What value of x makes this equation true? Show your reasoning.

Student Reflection:

Consider your work with functions over the last few lessons. What tool/format is most useful for you and your understanding? Why?

a. Graph b. Table c. Equation

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

Unlike talking, listening is a difficult thing to observe. At what points in the lesson did you observe students listening to one another's ideas today in class? What indicators do you have that they were listening?

Practice Problems

1. The cell phone plan from company C costs $10 per month, plus $15 per gigabyte for data used. The plan from company D costs $80 per month, with unlimited data.

   Rule C gives the monthly cost, in dollars, of using g gigabytes of data on company C's plan. Rule D gives the monthly cost, in dollars, of using g gigabytes of data on company D's plan.

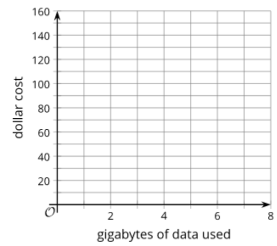
   a. Write a sentence describing the meaning of the statement $C(2) = 40$.

   b. Which value is smaller, $C(4)$ or $D(4)$? What does this mean for the two phone plans?

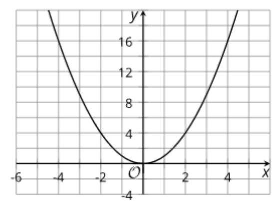
   c. Which value is smaller, $C(5)$ or $D(5)$? Explain how you know.

   d. For what number g is $C(g) = 130$?

   e. Draw the graph of each function.



2. Function g is represented by the graph.



   For what input value or values is $g(x) = 4$?

   a. 2
   b. -2 and 2
   c. 16
   d. none

3. Function P gives the perimeter of an equilateral triangle of side length s. It is represented by the equation $P(s) = 3s$.

   a. What does $P(s) = 60$ mean in this situation?

   b. Find a value of s to make the equation $P(s) = 60$ true.

4. Function W gives the weight of a puppy, in pounds, as a function of its age, t, in months.

   Describe the meaning of each statement.

   a. $W(2) = 5$

   b. $W(6) > W(4)$

   c. $W(12) = W(15)$

   (From Unit 5, Lesson 3)

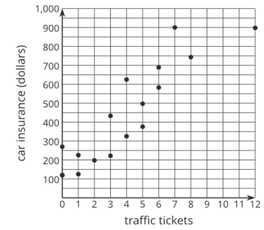
5. Function G takes a student's first name for its input and gives the number of letters in the first name for its output.

   a. Describe the meaning of $G(Jada) = 4$.

   b. Find the value of $G(Diego)$.

   (From Unit 5, Lesson 2)

6. A recent survey investigated the relationship between the number of traffic tickets a person received and the cost of the person's car insurance. The scatter plot displays the relationship. The line that models the data is given by the equation $y = 73x + 146.53$, where x represents the number of traffic tickets, and y represents the cost of car insurance.



   a. The slope of the line is 73. What does this mean in this situation? Is it realistic?

   b. The y-intercept is $(0, 146.53)$. What does this mean in this situation?

   (From Unit 4)

7. Diego is building a fence for a rectangular garden. It needs to be at least 10 feet wide and at least 8 feet long. The fencing he uses costs $3 per foot. His budget is $120. He wrote some inequalities to represent the constraints in this situation below.

   a. Explain what each equation or inequality represents.

Equation or inequalities	**What do they represent?**
$f = 2x + 2y$	
$x \geq 10$	
$y \geq 8$	
$3f \leq 120$	

   b. His mom says he should also include the inequality $f > 0$. Do you agree? Explain your reasoning.

   (From Unit 3)

8. Members of the band sold juice and popcorn at a college football game to raise money for an upcoming trip. The band raised $2,000. The amount raised is divided equally among the m members of the band.

   Which equation represents the amount, A, each member receives?

   a. $A = \frac{m}{2000}$
   b. $A = \frac{2000}{m}$
   c. $A = 2000m$
   d. $A = 2000 - m$

   (From Unit 2)

9. Answer the following questions:

   a. What is the five-number summary for 1, 3, 3, 3, 4, 8, 9, 10, 10, 17?

   b. When the maximum, 17, is removed from the data set, what is the five-number summary?

   (From Unit 1)

Lesson 6: Features of Graphs

PREPARATION

Lesson Goals	Learning Targets
• Analyze connections between statements that use function notation and features of graphs and describe (orally and in writing) these connections. • Interpret key features of a graph—the intercepts, maximums, minimums, and the intervals when the function is increasing or decreasing—in terms of a situation. • Understand and be able to use the terms "horizontal intercept," "vertical intercept," "maximum," and "minimum" when talking about graphs of functions.	• I can identify important features of graphs of functions and explain what they mean in the situations represented. • I understand and can use the terms "horizontal intercept," "vertical intercept," "maximum," and "minimum" when talking about functions and their graphs.

Lesson Narrative

Prior to this lesson, students have described characteristics of graphs, made sense of points on the graphs, and interpreted them in terms of a situation. In this lesson, students develop this work more formally, while continuing to use the idea of function as the focusing lens.

Students use mathematical terms such as **intercept**, **maximum**, and **minimum** in their graphical analyses, and relate features of graphs to features of the functions represented. For instance, they look at an interval in which a graph shows a positive slope and interpret that to mean an interval where the function's values are increasing. Students also use statements in function notation, such as $h(0)$ and $h(t) = 0$, to talk about key features of a graph.

By now, students are familiar with the idea of intercepts. A reminder that in these materials, the terms **horizontal intercept** and **vertical intercept** are used to refer to intercepts more generally, especially when a function is defined using variables other than x and y. If needed, clarify these terms for students who may be accustomed only to using x-intercept and y-intercept.

As students look for connections across representations of functions and relate them to quantities in situations, they practice making sense of problems (MP1) and reasoning quantitatively and abstractly (MP2). Using mathematical terms and notation to describe features of graphs and features of functions calls for attention to precision (MP6).

What math language will you want to support your students with in this lesson? How will you do that?

Focus and Coherence

Building On	Addressing
NC.8.F.5: Qualitatively analyze the functional relationship between two quantities. ● Analyze a graph determining where the function is increasing or decreasing; linear or non-linear. ● Sketch a graph that exhibits the qualitative features of a real-world function. **NC.M1.F-IF.2:** Use function notation to evaluate linear, quadratic, and exponential functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	**NC.M1.A-REI.10:** Understand that the graph of a two variable equation represents the set of all solutions to the equation. **NC.M1.F-IF.4:** Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; and maximums and minimums.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*15 minutes*)
- **Activity 2** (*10 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U5.L6 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Building On: NC.8.F.5

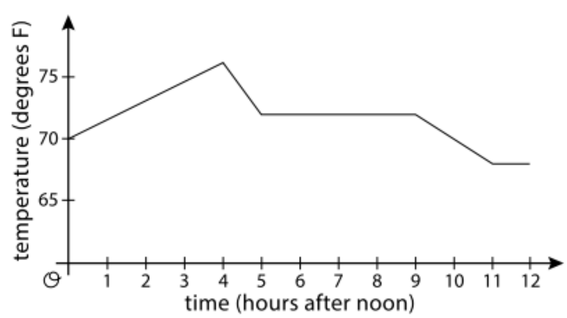
The purpose of this bridge is to provide students an opportunity to analyze a graph that represents a real-world scenario. Students will further analyze graphs in this lesson, using old and new vocabulary to describe key features of graphs. This task is aligned to question 6 in Check Your Readiness.

Student Task Statement

This graph shows the temperature in Diego's house between noon and midnight one day.[1]

Select **all** the true statements.

a. Time is a function of temperature.
b. The lowest temperature occurred between 4:00 and 5:00.
c. The temperature was increasing between 9:00 and 10:00.
d. The temperature was 74 degrees twice during the 12-hour period.
e. There was a 4-hour period during which the temperature did not change.



[1] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

DO THE MATH	PLANNING NOTES

Warm-up: Farmers Market (*5 minutes*)

Instructional Routine: Poll the Class	
Building On: NC.M1.F-IF.2	**Addressing:** NC.M1.A-REI.10

This warm-up is an opportunity to practice interpreting statements in function notation. It also draws attention to statements that correspond to the intercepts of a graph of a function (for instance, $d(0)$ and $d(m) = 0$), preparing students to reason about them in the lesson (particularly in the second activity).

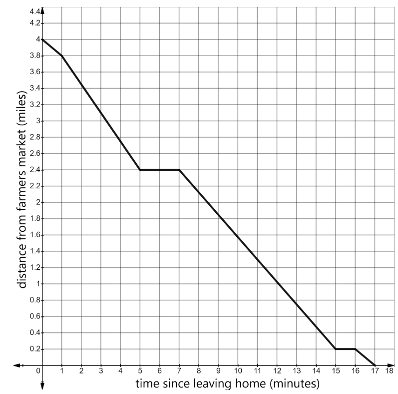
Step 1

- Ask students to interpret each statement in function notation before soliciting their response to each question.
- Make sure students understand, for instance, that $d(0)$ represents Diego's distance from the farmers market at the time of leaving (or at 0 minutes), and $d(m) = 0$ represents his distance from the farmers market being 0 km, m minutes after leaving his house. Also ensure students can articulate what they are solving for in, for instance, $d(12)$ and $d(m) = 2.4$.
- Provide students with 3 minutes of quiet time to respond to the questions, then *Poll the Class* for answers and resolve any discrepancies.

POLL THE CLASS	***What Is This Routine?*** This routine is used to register an initial response or an estimate, most often at the beginning of an activity or discussion. It can also be used when it is important to collect data from each student in class; for example, "What is the length of your ear in centimeters?" Every student in class reports a response to the prompt. Teachers need to develop a mechanism by which poll results are collected and displayed so that this frequent form of classroom interaction is seamless. Smaller classes might be able to conduct a roll call by voice. For larger classes, students might be given mini-whiteboards or a set of colored index cards to hold up. Free and paid commercial tools are also readily available. ***Why This Routine?*** Collecting data from the class to use in an activity with the *Poll the Class* routine makes the outcome of the activity more interesting. In other cases, going on record with an estimate makes people want to know if they were right and increases investment in the outcome. If coming up with an estimate is too daunting, ask students for a guess that they are sure is too low or too high. Putting some boundaries on possible outcomes of a problem is an important skill for mathematical modeling (MP4).

Student Task Statement

Noah and a sibling are going to make their favorite dinner. To find the ingredients, they take a Lyft ride from their home to Rosa Parks Farmers Market. This graph represents function d, which gives his distance from the farmers market, in miles, m minutes since leaving his home.



Use the graph to find or estimate:

1. $d(0)$
2. $d(12)$
3. the value of m when $d(m) = 2.4$
4. the value of m when $d(m) = 0$

Step 2

- If time permits, allow students to continue analyzing the graph and connecting to context with the following probing questions:
  - "Is the relationship between Noah's distance from the farmers market and time a linear function? How can we tell?" (No. The graph is not a line, which means the function's value changes at different rates.)
  - "Can we tell from the graph how far away Noah's house is from the farmers market? How?" (Yes. From the graph, we can see that at the time he leaves his house, he is 4 miles from the market.)
  - "Can we tell from the graph how long it took Noah to get to the market? How?" (Yes. From the graph, we can see the distance reaching 0 when the time is 17 minutes.)
  - "Does the graph slant upward or downward?" (Overall, the graph slants downward.)
  - "Why does the graph slant downward?" (As the input, m, increases, the output, $d(m)$, decreases.)
  - "Are there any times where the graph doesn't increase or decrease? What might be happening in those moments?" (Yes, when m is between 5 and 7 and when m is between15 and 16. These are the times when the Lyft isn't moving, perhaps because it was stopped at a light.)

DO THE MATH

PLANNING NOTES

Activity 1: A Toy Rocket and a Drone (*15 minutes*)

Instructional Routine: Co-Craft Questions (MLR5)
Addressing: NC.M1.F-IF.4

In this activity, students examine graphs of functions, identify and describe their key features, and connect these features to the situations represented. These key features include the horizontal and vertical intercepts, maximums and minimums, and intervals where a function is increasing or decreasing (or where a graph has a positive or a negative slope).

Neither the features nor the terms are likely new to students. The idea of intercepts was introduced in middle school and further developed in earlier units. Graphical features such as maximums and minimums have been considered intuitively in various cases. They are more precisely defined here. In a later activity, students will distinguish between a maximum of a graph and the maximum of a function.

Step 1

- Facilitate the *Co-Craft Questions* routine. Display the student task statement and graphs, without the questions.
  - Ask students to write down possible mathematical questions that could be asked about the situation.
  - Invite students to compare their questions before revealing the activity's questions.
  - Listen for and amplify any questions involving the behavior of the toy rocket and drone, or questions about features or places on the graph that show important information about each object's movement. This will help students produce the language of key features of graphs of functions, such as maximum, minimum, and intervals.
- Give students about 5 minutes of quiet work time. Follow with a whole-class discussion.

RESPONSIVE STRATEGY

To help get students started, display sentence frames such as, "It looks like. . .", "I notice that. . .", "____ represents ____.", "What does this part of ____ mean?"

Supports accessibility for: Language; Organization

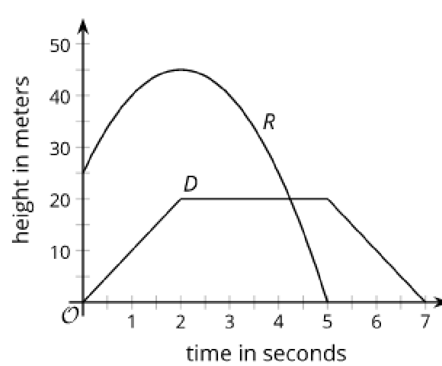
Advancing Student Thinking: When analyzing the graphs and describing what is happening with each object, some students may mistakenly think that the horizontal axis represents horizontal distance, neglecting to notice that it represents time. They may then describe how the objects were moving vertically as they traveled horizontally, rather than with respect to the number of seconds since they took off. Encourage these students to check the label of each axis and revisit their descriptions.

Student Task Statement

A toy rocket and a drone were launched at the same time.

Here are the graphs that represent the heights of two objects as a function of time since they were launched.

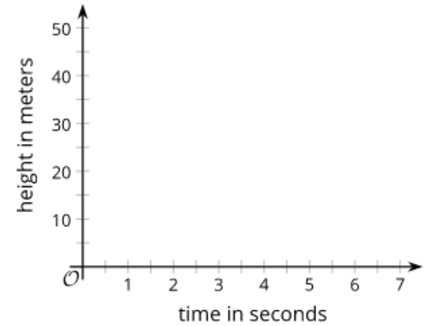
Height is measured in meters above the ground and time is measured in seconds since launch.



1. Analyze the graphs and describe—as precisely as you can—what was happening with each object. Your descriptions should be complete and precise enough that someone who is not looking at the graph could visualize how the objects were behaving.
2. Which parts or features of the graphs show important information about each object's movement? List the features or mark them on the graphs.

Step 2

- Select a few students to share their description of the graphs, and have each student describe the motion of one flying object.
- Display a blank coordinate plane for all to see. As each student shares their response, sketch a graph to match what is being described.
- For any gaps in their description, make assumptions and sketch accordingly. (For example, if a student states that the toy rocket reaches a height of 45 meters after 2 seconds but does not state its starting height, start the curve at $(0,0)$, $(0,40)$, or any other point besides $(0,25)$.) If requested, allow students to refine their descriptions and adjust the sketch accordingly.



Step 3

- Invite other students to share their response to the last question. On the graphs, highlight the features students noted. Use the terms “vertical intercepts,” “horizontal intercepts,” “maximum,” and “minimum” to refer to those features and label them on the graphs.
- Explain to students that:
  - A point on the graph that is as high as or higher than all other points is called a maximum of the graph.
  - A point on the graph that is as low as or lower than all other points is called a minimum of the graph.
  - A graph could have more than one maximum or minimum. For instance, the points $(2, D(2))$ and $(5, D(5))$ are both maximums, and $(0, D(0))$ and $(7, D(7))$ are both minimums.
- If no students mentioned the intervals in which each function was increasing, staying constant, or decreasing, draw their attention to these features on the graphs and label them as such.

DO THE MATH	PLANNING NOTES

Activity 2: The Jump (*10 minutes*)

Instructional Routine: Discussion Supports (MLR8)	
Addressing: NC.M1.F-IF.4	

Earlier in the lesson, students identified key features of a graph of a function and related them to the features of a situation. In this activity, students continue to connect graphical and verbal representations of a function, applying the mathematical terms they learned. They also connect each feature (described in words and on the graph) to an expression or equation that could represent it mathematically, written in function notation.

During Step 2, students learn to distinguish between a maximum or minimum of a graph and the maximum or minimum of a function. Students see that a maximum of a graph refers to a point on a graph that is as high or higher than all other points, while the maximum of a function is a function value that is equal to or greater than all other values of that function. They are cautioned that a graph may not reveal the true maximum or minimum value of a function, since graphs may only display input-output pairs of a function on a certain interval: for example, over a certain period of time.

Step 1

- Ask students to arrange students themselves in pairs or use visibly random grouping.
- Give students a few minutes of quiet think time and then time to discuss their response with their partner.

Use *Discussion Supports* to support small-group discussion. As students share their matches and explain their reasoning to their partner, display the following sentence frames for all to see:

- "_____ matches _____ because..."
- "I noticed _____ , so I matched..."
- "The strategy that I used was..."

RESPONSIVE STRATEGY

Use color coding and annotations to highlight connections between representations in a problem. For example, invite students to highlight matching descriptions, equations, and parts of the graph in the same color.

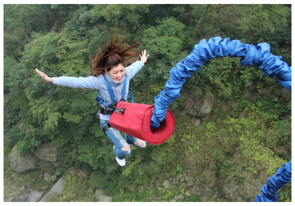
Supports accessibility for: Visual-spatial processing

- Encourage students to challenge each other when they disagree. While monitoring discussions, amplify student ideas to demonstrate use of mathematical language such as "first peak," "maximum," "minimum," or "vertical intercept." This routine will help students connect graphical and verbal representations of a function through partner discussions.

Monitoring Tip: As students discuss with their partners, monitor the language students use to form their matches, especially the one without a verbal description. Identify a few students to share out with the whole group in Step 2, and let them know that they may be asked to share later. Include at least one student who does not typically volunteer.

Student Task Statement

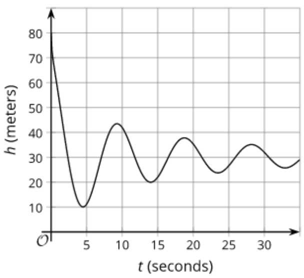
In a bungee jump, the height of the jumper is a function of time since the jump begins.



Function h defines the height, in meters, of a jumper above a river, t seconds since leaving the platform.

Here is a graph of function h, followed by five expressions or equations and five graphical features.

Expressions or equations	Features of graph
$h(0)$ $h(t) = 0$ $h(4)$ $h(t) = 80$ $h(t) = 45$	• First dip in the graph • Vertical intercept • First peak in the graph • Horizontal intercept • Maximum



Match each description of the jump to a corresponding expression or equation and to a feature on the graph. One expression or equation does not have a matching verbal description. Its corresponding graphical feature is also not shown on the graph. Interpret that expression or equation in terms of the jump and in terms of the graph of the function. Record your interpretation in the last row of the table.

Description of jump	Expression or equation	Feature of graph
a. The greatest height that the jumper is from the river		
b. The height from which the jumper was jumping		
c. The time at which the jumper reached the highest point after the first bounce		
d. The lowest point that the jumper reached in the entire jump		
e. ______________________		

Are You Ready For More?

Based on the information available, how long do you think the bungee cord is? Make an estimate and explain your reasoning.

Step 2

- Select students to share how they made their matches and how they interpreted the expression or equation without a matching verbal description. After each student shares their thinking, ask if others also approached it the same way.
- Make sure students can interpret $h(t) = 0$ to mean that the jumper is no longer in the air and is in fact on the surface of the water. Because the graph has no horizontal intercept, and because no verbal descriptions to this effect were given, we know $h(t) = 0$ had no match.
- This is a good time to caution the students that a graph may not display all the input-output pairs of a function, and therefore it's possible that the maximum or minimum of a function may not be displayed on the graph. Ask students:
  - "What is the greatest value of function h?" (80)
  - "How do we know that 80 is the greatest, or that $h(t)$ could not have greater values?" (The jumper could not be higher than the jumping platform.)
  - "What is the least value of function h?" (about 10, based on the graph)
  - "How do we know that 10 is the least value, or that $h(t)$ could not have lesser values?" (We don't. After 40 seconds, the jumper could go lower, say, if lowered into a receiving boat or if released into the river.)
- Explain that 10 is the minimum value of the function h on the interval $t = 0$ *to* $t = 40$. That is, we know for sure that between 0 and 40 seconds, 10 is the lowest value that $h(t)$ takes on. If we expand the interval to include later times, it's possible that $h(t)$ may take on a lower value.
- Emphasize that a maximum (or minimum) of a graph is a *point*. The maximum (or minimum) of a function, however, is a *value* that is the greatest (or least) for any input.

DO THE MATH

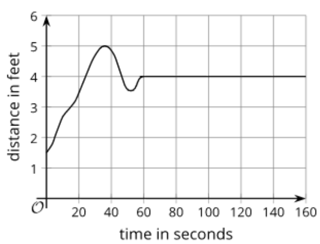
PLANNING NOTES

Lesson Debrief (*5 minutes*)

Instructional Routine: Take Turns

The purpose of this lesson is to have students interpret key features of a graph in terms of a situation and to practice using the associated precise vocabulary, specifically horizontal and vertical intercepts, maximums and minimums, and intervals of increase and decrease.

Engage students in the *Take Turns* routine as part of the debrief of this lesson. Choose what questions will be prioritized in the full class discussion. Consider providing students with whiteboards to share their sketches.



Keep students in pairs. Display the graph and the descriptions of two functions for all students to see:

- Function b gives the vertical distance (or the height) of a bee from the ground as a function of time, t.
- Function d gives the distance of a child from where his mom is sitting as a function of time, t.

Tell students that both functions have the same graph, and the outputs of both functions are measured in feet and the inputs are both measured in seconds.

Ask partners to choose a function. For that function, they should *Take Turns* identifying the following features on the graph and interpreting them in terms of the situation:

- vertical intercept
- horizontal intercept
- maximum
- minimum
- intervals where the function is increasing
- intervals where the function is decreasing
- intervals where the function is staying constant
- the value of t when $b(t) = 3.5$ or $d(t) = 3.5$

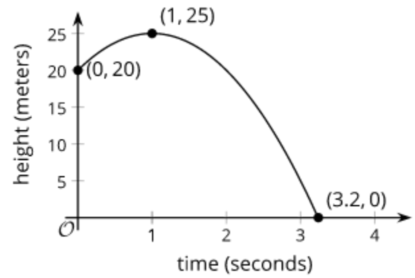
Then, discuss questions such as:

- "How can you tell that a point on the graph is a maximum, a minimum, or neither?" (Look around it, left and right, to see whether the given point is as high or higher than all the other points, as low or lower than all the others, or neither.)
- "How many intercepts can the graph of a function have?" (An unlimited number on the horizontal axis, but only one on the vertical axis.)

PLANNING NOTES

Student Lesson Summary and Glossary

The graph of the function can give us useful information about the quantities in a situation. Some points and features of a graph are particularly informative, so we pay closer attention to them.



Let's look at the graph of function h, which gives the height, in meters, of a ball t seconds after it is tossed up in the air. From the graph, we can see that:

- The point $(0, 20)$ is the **vertical intercept** of the graph, or the point where the graph intersects the vertical axis.

  This point tells us that the initial height of the ball is 20 meters, because when t is 0, the value of $h(t)$ is 20.

  The statement $h(0) = 20$ captures this information.

  > **Vertical intercept**: The point where a graph crosses the vertical axis, so its coordinates have the form $(0, b)$. In the graph of a function, b represents the output for an input of 0. If the axis is labeled with the variable y, the vertical intercept is also called the y-intercept.
  >
  > The term is sometimes used to mean just the y-coordinate of the point where the graph crosses the vertical axis. The vertical intercept of the graph of $y = 3x - 5$ is $(0, -5)$, or just -5.

- The point $(1, 25)$ is the highest point on the graph, so it is a **maximum** of the graph.

  The value 25 is also the maximum value of the function h. It tells us that the highest point the ball reaches is 25 feet, and that this happens 1 second after the ball is tossed.

  > **Maximum**: A value of the function that is greater than or equal to all the other values. The maximum of the graph of the function is the corresponding highest point on the graph.

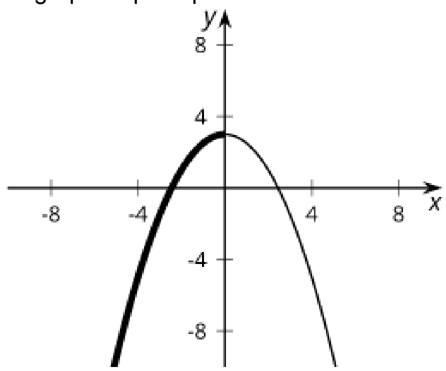
- The point $(3.2, 0)$ is a **horizontal intercept** of the graph, a point where the graph intersects the horizontal axis. This point is also the lowest point on the graph, so it represents a **minimum** of the graph. This tells us that the ball hits the ground 3.2 seconds after being tossed up, so the height of the ball is 0 when t is 3.2, which we can write as $h(3.2) = 0$. Because h cannot have any lower value, 0 is also the minimum value of the function.

  > **Horizontal intercept**: The point where a graph crosses the horizontal axis, so its coordinates have the form $(a, 0)$. In the graph of a function, a is the input value that results in an output of 0. If the axis is labeled with the variable x, the horizontal intercept is also called the x-intercept. The term is sometimes used to refer only to the x-coordinate of the point where the graph crosses the horizontal axis.
  >
  > **Minimum**: A value of the function that is less than or equal to all the other values. The minimum of the graph of the function is the corresponding lowest point on the graph.

- The height of the graph increases when t is between 0 and 1. Then, the graph changes direction and the height decreases when t is between 1 and 3.2. Neither the **increasing** part nor the **decreasing** part is a straight line. This means that the ball increases in height in the first second after being tossed, and then falls between 1 second and 3.2 seconds. It also tells us that the height does not increase or decrease at a constant rate.

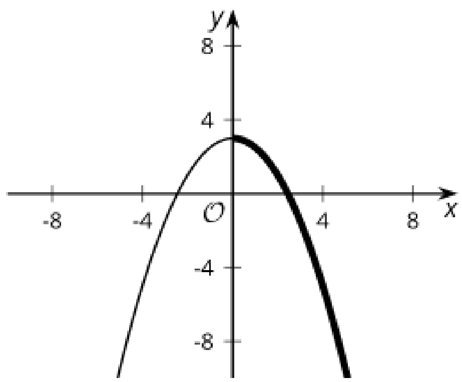
Increasing (function): A function is increasing if its outputs get larger as the inputs get larger, resulting in an upward sloping graph as you move from left to right.

A function can also be increasing just for a certain interval. For example the function f given by $f(x) = 3 - x^2$, whose graph is shown, is increasing for $x \leq 0$ because the graph slopes upward to the left of the vertical axis.



Decreasing (function): A function is decreasing if its outputs get smaller as the inputs get larger, resulting in a downward sloping graph as you move from left to right.

A function can also be decreasing just for a certain interval. For example the function f given by $f(x) = 3 - x^2$, whose graph is shown, is decreasing for $x \geq 0$ because the graph slopes downward to the right of the vertical axis.



Cool-down: The Squirrel (*5 minutes*)

Addressing: NC.M1.F-IF.4

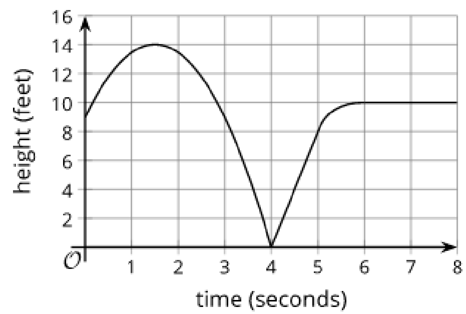
Cool-down Guidance: More Chances
Students will have many more opportunities to understand the mathematical ideas in this cool-down, so there is no need to slow down or add additional work to the next lessons. Instead, use the results of this cool-down to provide guidance for what to look for and emphasize over the next several lessons to support students in advancing their current understanding.

Cool-down

A squirrel runs up and down a tree.

The graph shows the height of the squirrel, h (feet), as a function of time, t (seconds).

1. What is the highest point the squirrel reaches?
2. Solve $h(t) = 0$. What does this solution tell you about the squirrel?
3. Find the vertical intercept of the graph. What does it tell you about the squirrel?



Student Reflection:

Describe your favorite mistake made today during class. How did you correct it? What did the way you handled it tell you about yourself?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

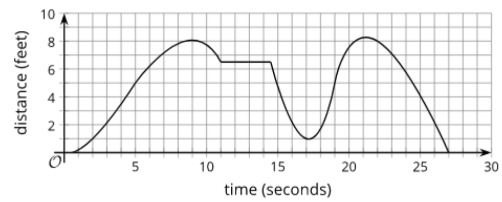
TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

What was your favorite mistake you observed today? How might you address it in the upcoming lesson to close the gaps?

Practice Problems

1. This graph represents Andre's distance from his bicycle as he walks in a park.



Decide whether the following statements are **true** or **false**.

   a. The graph has multiple horizontal intercepts.

   b. A horizontal intercept of the graph represents the time when Andre was with his bike.

   c. A minimum of the graph is $(17, 1)$.

   d. The graph has two maximums.

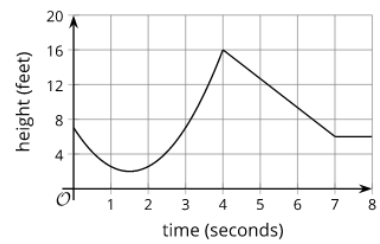
   e. About 21 seconds after he left his bike, he was the farthest away from it, at about 8.3 feet.

2. The graph represents the temperature in degrees Fahrenheit as a function of time.

   a. Tell the story of the temperature throughout the day.

   b. Identify the maximum and minimum of the function and where the function is increasing and decreasing.

3. Match each feature of the situation with a corresponding statement in function notation.



a. maximum height b. minimum height c. height staying the same d. starting height	i. $h(0) = 7$ ii. $h(1.5)$ iii. $h(4)$ iv. $h(t) = 6$ for $7 \le t \le 8$

4. Here are the equations that define three functions.

$$f(x) = 4x - 5$$
$$g(x) = 4(x - 5)$$
$$h(x) = \frac{x}{4} - 5$$

   a. Which function value is the largest: $f(100)$, $g(100)$, or $h(100)$?

   b. Which function value is the largest: $f(-100)$, $g(-100)$, or $h(-100)$?

   c. Which function value is the largest: $f(\frac{1}{100})$, $g(\frac{1}{100})$, or $h(\frac{1}{100})$?

(From Unit 5, Lesson 4)

5. Function f is defined by the equation $f(x) = x^2$.

   a. What is $f(2)$?

   b. What is $f(3)$?

   c. Explain why $f(2) + f(3) \neq f(5)$.

(From Unit 5, Lesson 4)

6. A sports journalist is trying to rank the eight best men's college basketball teams from the 2020–21 season in the Atlantic Coast Conference based on the number of games each team won last year against opponents in the conference. The table shows the colleges for several different win totals.

Games won	13	11	9	11	10	10	8	9
College team	Virginia	Florida State	Virginia Tech	Georgia Tech	Clemson	North Carolina	Louisville	Syracuse

   a. What does the independent variable in the relationship represent?

   b. What does the dependent variable in the relationship represent?

   c. Is the relationship a function? Why or why not?

   (From Unit 5, Lesson 1)

7. On July 7, 2021, the hourly temperatures in Charlotte, in degrees Fahrenheit, were:

Hour	7 a.m.	8 a.m.	9 a.m.	10 a.m.	11 a.m.	12 p.m.	1 p.m.	2 p.m.	3 p.m.
Temperature	71	75	78	82	85	87	88	89	90

   a. Use technology to determine the line of best fit. Use $x = 0$ to represent 7 a.m.

   b. Elena says that at 10:00 p.m., the temperature should be about 104 degrees. Do you agree with Elena? Why or why not?

   (From Unit 4)

8. Find the equation for a line perpendicular to $y = 3x - 7$ that passes through the origin.

   (From Unit 3)

9. Priya bought two plants for a science experiment. When she brought them home, the first plant was 5 cm tall, and the second plant was 4 cm. Since then, the first plant has grown 0.5 cm a week, and the second plant has grown 0.75 cm a week.

   a. Which plant is taller at the end of 2 weeks? Explain your reasoning.

   b. Which plant is taller at the end of 10 weeks? Explain your reasoning.

   c. Priya represents this situation with the equation $5 + 0.5w = 4 + 0.75w$, where w represents the end of week. What does the solution to this equation, $w = 4$, represent in this situation?

   d. What does the solution to the inequality $5 + 0.5w > 4 + 0.75w$ represent in this situation?

   (From Unit 2)

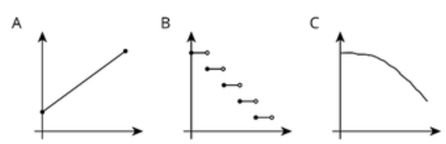
10. Match the graphs to the following situations (you can use a graph multiple times). For each match, name possible independent and dependent variables and how you would label the axes.[2]

   a. Tyler pours the same amount of milk from a bottle every morning.

   b. A plant grows the same amount every week.

   c. The day started very warm but then it got colder.

   d. A carnival has an entry fee of $5, and tickets for rides cost $1 each.



   (Addressing NC.8.F.5)

[2] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

Lesson 7: Using Graphs to Find Average Rate of Change

PREPARATION

Lesson Goals	Learning Targets
• Given a graph of a function, estimate or calculate the average rate of change over a specified interval. • Recognize that the slope of a line joining two points on a graph of a function is the average rate of change. • Understand that the average rate of change describes how much the output of a function changes for every unit of change in the input.	• When given a graph of a function, I can estimate or calculate the average rate of change between two points. • I understand the meaning of the term "average rate of change."

Lesson Narrative

Previously, students have characterized how functions are changing qualitatively by describing them as increasing, staying constant, or decreasing in value. In earlier units and prior to this course, students have also computed and compared the slopes of linear graphs and interpreted them in terms of rates of change. In this lesson, students learn to characterize changes in functions quantitatively, by using average rates of change.

Students learn that **average rate of change** can be used to measure how much a function changes over a given interval. This can be done when we know the input-output pairs that mark the interval of interest or by estimating them from a graph.

Attention to units is important in computing or estimating average rates of change because units give meaning to how much the output quantity changes relative to the input. In thinking carefully about appropriate units to use, students practice attending to precision (MP6).

Students also engage in aspects of mathematical modeling (MP4) when they use a data set or a graph to compute average rates of change and then use it to analyze a situation or make predictions.

What is the main purpose of this lesson? What is the one thing you want your students to take away from this lesson?

Focus and Coherence

Building On	Addressing
NC.8.F.4: Analyze functions that model linear relationships. • Understand that a linear relationship can be generalized by $y = mx + b$. • Write an equation in slope-intercept form to model a linear relationship by determining the rate of change and the initial value, given at least two (x, y) values or a graph. • Construct a graph of a linear relationship given an equation in slope-intercept form. • Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of the slope and y-intercept of its graph or a table of values.	**NC.M1.F-IF.6:** Calculate and interpret the average rate of change over a specified interval for a function presented numerically, graphically, and/or symbolically.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*15 minutes*)
- **Activity 2** (*10 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U5.L7 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Building On: NC.8.F.4

This bridge provides students with the opportunity to interpret the rate of change and initial value of a linear function that models a car traveling down a hill. This task connects students' work with rate of change in grade 8 to their work in this lesson on average rate of change. This task is aligned to question 5 in Check Your Readiness.

Student Task Statement

A function assigns to the inputs shown the corresponding outputs given in the table.[1]

1. Do you suspect the function is linear? Compute the rate of change of this data for at least three pairs of inputs and their corresponding outputs.
2. What equation seems to describe the function?
3. As you did not verify that the rate of change is constant across **all** input/ output pairs, check that the equation you found in problem 1 does indeed produce the correct output for each of the four inputs 1, 2, 4, and 6.
4. What will the graph of the function look like? Explain.

Input	Output
1	2
2	-1
4	-7
6	-13

[1] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education, which was originally developed and authored by Great Minds. It is licensed under the Creative Commons Attribution-NonCommercial-ShareAlike 3.0 United States (CC BY-NC-SA 3.0 US).

DO THE MATH

PLANNING NOTES

Warm-up: Temperature Drop (*5 minutes*)

Instructional Routine: Poll the Class	
Building Towards: NC.M1.F-IF.6	

In this warm-up, students compare the changes in output (temperature) over two intervals of input (time). The temperature in one interval changes by a greater amount than in the other interval, but in the latter, temperature changes more rapidly.

Thinking about what it means for temperature to drop "faster" activates the idea of rates of change and prepares for the work later in the lesson.

Step 1

- Display the task for all to see.
- Give students 2 minutes of quiet time to work on the question.

Advancing Student Thinking: Some students may ask for a clarification as to what is meant by "faster" in this situation. Acknowledge that thinking about its meaning in context is a great way to approach the task. Encourage these students to interpret the word based on their understanding of the given information.

Student Task Statement

Here are the recorded temperatures at three different times on a winter evening.

- Tyler says the temperature dropped faster between 4 p.m. and 6 p.m.
- Mai says the temperature dropped faster between 6 p.m. and 10 p.m.

Time	4 p.m.	6 p.m.	10 p.m.
Temperature	$25°F$	$17°F$	$8°F$

Who do you agree with? Explain your reasoning.

Step 2

- *Poll The Class* on whether they agree with Mai or with Tyler. Select one or two students from each group to explain their reasoning. As they explain, record and display their thinking for all to see.
- After both groups have had a chance to present, ask if anyone changed their mind because of the explanation they heard. If so, invite them to share their reasons.
- It is not necessary to resolve the question at this point. Students will continue thinking about this question in the next activity.

DO THE MATH	PLANNING NOTES

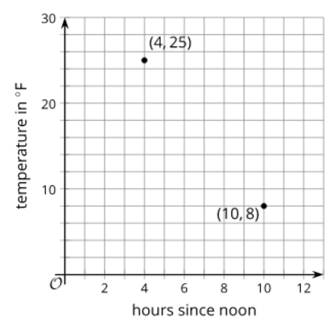
Activity 1: Drop Some More (*15 minutes*)

Instructional Routines: Notice and Wonder; Compare and Connect (MLR7) - Responsive Strategy
Addressing: NC.M1.F-IF.6

This activity introduces students to average rate of change by building on what students know about rate of change and slope. (Note that some students may have been briefly introduced to this concept in Station C (Laptop Battery Charge) in the Checkpoint lesson of the previous unit.)

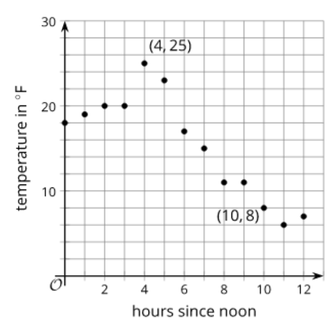
Students see that finding the change in the output for every unit of change in the input can be a useful way to generalize what happens between two function values, regardless of the behaviors of individual data points between them. They recognize that this number tells us how, *on average*, one quantity is changing relative to the other, and that it can be useful for comparing the trends in different intervals of a function. In the activity, students find average rates of change by reasoning and using their knowledge of linear relationships. In the whole-group discussion of Step 4, students generalize their reasoning and see that finding the average rate of change is indeed equivalent to finding the slope of the line connecting the two points.

Step 1



- Without students looking at their materials, display a scatter plot with the two temperature data points plotted, as shown.
- Tell students that it shows the temperature at 4 p.m. and the temperature at 10 p.m.
- Ask students:
  - "How did the temperature change between 4 p.m. and 10 p.m.?" (It fell.)
  - "How much did it fall?" ($17°F$)
  - "Can we tell how fast, or at what rate, the temperature was falling?"
    - "If so, how?" (It fell $17°F$ in 6 hours. If it was falling at a constant rate, it would be falling $\frac{17}{6}$ or about $2.8°F$ per hour. If we connect the points with a line and find its slope, it would be about -2.8.)
    - "If not, why not?" (There is not enough information about what happened in between the two data points. It might not be falling at the same rate.)

Step 2



- Display a scatter plot with the hourly data points plotted.

Give students a moment to *Notice and Wonder* something about the data and solicit a few responses. If no students mention the temperatures changing at different rates (a lot, a little, or not at all) each hour, ask them about it.

- Discuss with students: “Do the hourly data between 4 p.m. and 10 p.m. help us better characterize how fast the temperature was falling between 4 p.m. and 10 p.m.? Could we still say that it was falling about $2.8°F$ an hour?”
- Students are likely to comment that a line connecting $(4, 25)$ and $(10, 8)$ approximates the distribution of points in that interval and conclude that $-2.8°F$ per hour is a reasonable rate to use.
- Explain to students that it can be helpful to have a way to quantify how a quantity is changing over a particular interval, without worrying about the smaller changes in between them. The rate of $-2.8°F$ per hour serves that purpose. It is the average rate of change between 4 p.m. and 10 p.m.

Step 3

Provide students with quiet work time to read the student task statement and respond to the questions.

Monitoring Tip: As students work, monitor for the following strategies (the examples shown are for finding the average rate of change between 6 p.m. and 10 p.m.):

- Find the hourly changes in temperature and calculate its average. The temperature fell by $2°F$ the first hour, then $4°F$, $0°F$, and $3°F$ in the next three hours. The average is $\frac{2+4+0+3}{4} = \frac{9}{4}$, or $2.25°F$ per hour.
- Find the overall change in temperature and divide by the overall change in hours. The temperature fell $9°F$ in 4 hours, which means an average drop of $2.25°F$ per hour.
- Find the slope of the line connecting the two end points of the interval. The slope between $(6, 17)$ and $(10, 8)$ is $\frac{8-17}{10-6} = \frac{-9}{4} = -2.25$, which means an average drop of $2.25°F$ per hour.

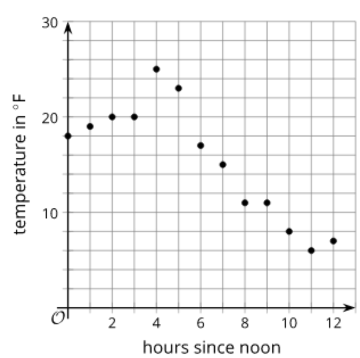
Identify one or two students who use each strategy and let them know that they may be asked to share later. Include at least one student who does not typically volunteer.

Student Task Statement

The table and graphs show a more complete picture of the temperature changes on the same winter day. The function T gives the temperature in degrees Fahrenheit, h hours since noon.

1. Find the average rate of change for the following intervals. Explain or show your reasoning.
    a. between noon and 1 p.m.
    b. between noon and 4 p.m.
    c. between noon and midnight
2. Remember Mai and Tyler’s disagreement? Use average rate of change to show which time period—4 p.m. to 6 p.m. or 6 p.m. to 10 p.m.—experienced a faster temperature drop.

h	$T(h)$
0	18
1	19
2	20
3	20
4	25
5	23
6	17
7	15
8	11
9	11
10	8
11	6
12	7

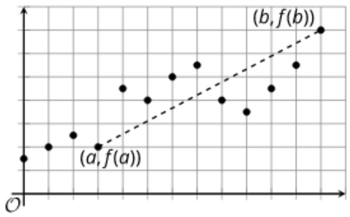


Are You Ready For More?

1. Over what interval did the temperature decrease the most rapidly?
2. Over what interval did the temperature increase the most rapidly?

Step 4

- Call on selected students to share how they determined the average rates of change for the specified intervals. Arrange the presentations in the order listed in the Monitoring Tip.



- Point out that all strategies involve finding how much the temperature (the output) changed per unit of change in time (the input), and that this is done by division.
- Display a graph with two points labeled $(a, f(a))$ and $(b, f(b))$. Ask students how we might find the average rate of change between the two points.
- Make sure students can generalize their work and see that:

average rate of change $= \frac{f(b)-f(a)}{b-a}$

RESPONSIVE STRATEGY

Instead of the whole-class discussion in Step 4, invite students to create a visual display that shows how they determined the average rates of change for the specified intervals. Students should consider how to represent their strategy so that other students will be able to understand their solution method. Students may wish to add notes or details to their displays to help communicate their reasoning and thinking. Begin the whole-class discussion by selecting and arranging 2–4 displays for all to see. Give students 1–2 minutes of quiet think time to interpret the displays before inviting the authors to share their strategies as described in the activity narrative. This will help students make connections between different ways to find the average rate of change.

Compare and Connect (MLR7)

DO THE MATH	PLANNING NOTES

Activity 2: Populations of Two States (*10 minutes*)

Instructional Routine: Stronger and Clearer Each Time (MLR1)
Addressing: NC.M1.F-IF.6

This activity allows students to practice finding and interpreting average rates of change in a different context. No tables of values are given here, so students will need to estimate the coordinates on the graph to compute the average rates of change.

Students reason quantitatively and abstractly (MP2) as they extract contextual information from a graph, manipulate it symbolically, and then interpret the numerical results in context. To interpret the average rates of change, students need to pay attention to the units used to measure the input (years) and output (people in millions). This is a chance to practice attending to precision (MP6).

RESPONSIVE STRATEGIES

Create a display of important terms and vocabulary. During Step 1, take time to review terms that students will need to access this activity. Invite students to suggest language or diagrams to include that will support their understanding of rate of change, slope, and average rate of change.

Supports accessibility for: Conceptual processing; Language

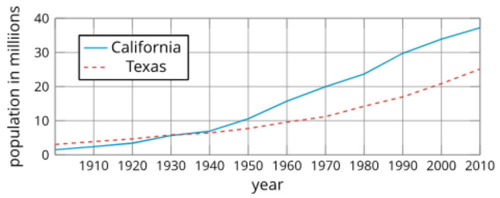
Step 1

- Provide students with quiet work time to read the student task statement and respond to the questions.

Advancing Student Thinking: When calculating average rates of change for California and Texas between 1970 and 2010, students are likely to get slightly different results from one another. Discuss why this might be. If not mentioned by students, point out that it is most likely due to differences in how they estimated the populations from the graphs.

Student Task Statement

The graphs show the populations of California and Texas over time.



1.
   a. Estimate the average rate of change in the population in each state between 1970 and 2010. Show your reasoning.

   b. In this situation, what does each rate of change mean?

2. Which state's population grew more quickly between 1900 and 2000? Show your reasoning.

Step 2

- Use the *Stronger and Clearer Each Time* routine to help students improve their verbal and written responses to Question 2. Give students time to meet with two or three partners to share and get feedback on their responses. The listener should press for detail and precision in language by asking "Can you say that a different way?"; "Does your method work using a different time interval?"; or "Besides computing the average rate of change, can you think of another way to determine this?"
- Invite students to go back and revise or refine their written explanation based on the feedback from peers. This will help students find and interpret average rates of change in context.

STRONGER AND CLEARER EACH TIME	***What Is This Routine?*** Students write a first draft response to a prompt, then engage in successive pair-shares to have multiple opportunities to refine and clarify their response through conversation, and then finally revise their original response. Throughout this process, students should be encouraged to press each other for clarity and details. ***Why This Routine?*** *Stronger and Clearer Each Time* (MLR1) provides a structured and interactive opportunity for students to revise and refine both their ideas and their verbal and written output. The routine provides a purpose for student conversation and fortifies student output.

Step 3

- Invite students to share their response to the last question. Make sure students recognize that it is not always necessary to compute average rates of change or slopes of lines to compare the trends of two functions. We can visually compare the steepness of the lines connecting the endpoints of each graph.

DO THE MATH	PLANNING NOTES

Lesson Debrief (*5 minutes*)

The purpose of this lesson is to introduce students to the concept of average rate of change, and to be able to connect that concept to slope of a line and steepness on a graph. The goal of this debrief is to have students articulate what the average rate of change of a function over a specified interval represents.

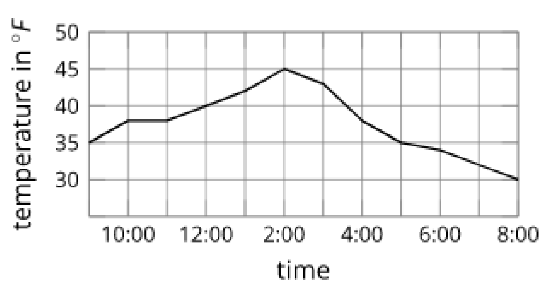
Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion. Discuss questions such as:

- "How would you explain 'average rate of change' to a classmate who is absent today? What does it tell us? How do we find it?"
- "Which representation—a table or a graph—can give us a better sense of the general trend of a function over a certain interval? Why?" (A graph, because we can visualize a line that fits the data in an interval and reason about its slope.)
- "What does a negative average rate of change tell us?" (On average, the output of the function is decreasing for every unit of increase in input.)
- "If a function has a negative average rate of change over an interval, does it mean that the function value never increases?" (No, there might be parts of the interval where the function value rises, but overall, it is falling.) If needed, refer to the graph in Step 4 of Activity 2 for an example of a function with a positive average rate of change over an interval which also decreases over parts of the interval.
- "When dealing with Mai and Tyler's case, we compared average rates of change over two intervals that are not the same length. Was that a fair comparison? Does the length of the interval matter?" (Yes, it is a fair comparison. The length of the interval doesn't matter because an average rate of change gives the amount of change per unit of input.)

PLANNING NOTES

Student Lesson Summary and Glossary

Here is a graph of one day's temperature as a function of time.



The temperature was $35°F$ at 9 a.m. and $45°F$ at 2 p.m., an increase of $10°F$ over those 5 hours.

The increase wasn't constant, however. The temperature rose from 9 a.m. and 10 a.m., stayed steady for an hour, then rose again.

- On average, how fast was the temperature rising between 9 a.m. and 2 p.m.?

  Let's calculate the **average rate of change** and measure the temperature change per hour. We do that by finding the difference in the temperature between 9 a.m. and 2 p.m. and dividing it by the number of hours in that interval.

  average rate of change $= \frac{45-35}{5} = \frac{10}{5} = 2$

  On average, the temperature between 9 a.m. and 2 p.m. increased $2°F$ per hour.

- How quickly was the temperature falling between 2 p.m. and 8 p.m.?

  average rate of change $= \frac{30-45}{6} = \frac{-15}{6} = -2.5$

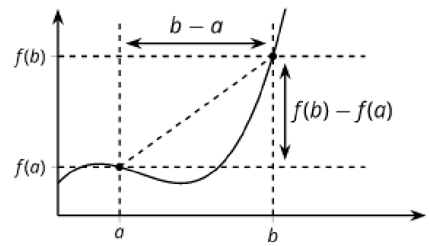
  On average, the temperature between 2 p.m. and 8 p.m. dropped by $2.5°F$ per hour.

In general, we can calculate the average rate of change of a function f, between input values a and b, by dividing the difference in the outputs by the difference in the inputs.

average rate of change $= \frac{f(b)-f(a)}{b-a}$

If the two points on the graph of the function are $(a, f(a))$ and $(b, f(b))$, the average rate of change is the slope of the line that connects the two points.

Average rate of change: The average rate of change of a function f between inputs a and b is the change in the outputs divided by the change in the inputs: $\frac{f(b)-f(a)}{b-a}$. It is the slope of the line joining $(a, f(a))$ and $(b, f(b))$ on the graph.



Cool-down: Population of a City (*5 minutes*)

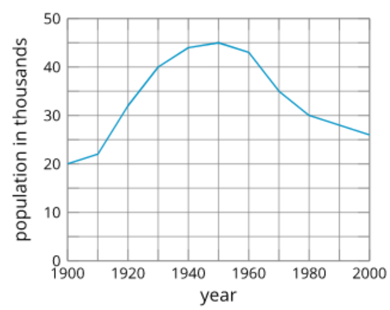
Addressing: NC.M1.F-IF.6

Cool-down Guidance: Points to Emphasize
Student errors on this cool-down are a great opportunity to highlight the importance of using the numbers on the axes rather than just counting, and also the importance of thinking about context when stating the rate of change. You might address errors after Activity 1 in Lesson 8 by showing a few examples of student work from this cool down to address misconceptions. There are also more opportunities in Unit 6 to review this concept.

Cool-down

1. The graph shows the population of a city from 1900 to 2000.

   What is the average rate of change of the population between 1930 and 1950? Show your reasoning.



2. For each interval, decide if the average rate of change is positive or negative, and check the appropriate box.

Interval	Positive	Negative
a. From 1930 to 1940		
b. From 1950 to 1970		
c. From 1930 to 1970		

3. In which decade (10-year interval) did the population grow the fastest? Explain how you know.

Student Reflection:

Consider the ways in which you have engaged with your peers while learning about functions. How does collaborating in math class help us learn? How has it helped you learn, personally?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

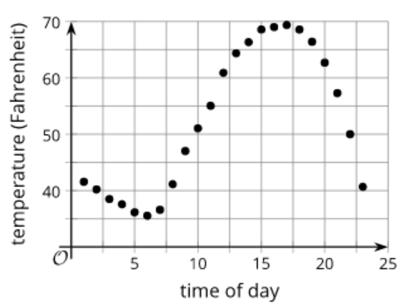
TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

How did the work of the previous lesson lay the foundation for students to be successful in Activity 1 of this lesson?

Practice Problems

1. The temperature was recorded at several times during the day. Function T gives the temperature in degrees Fahrenheit, n hours since midnight. Here is a graph for this function.



   For each time interval, decide if the average rate of change is positive, negative, or zero.

   a. From $n = 1$ to $n = 5$

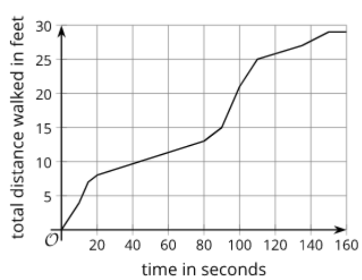
   b. From $n = 5$ to $n = 7$

   c. From $n = 10$ to $n = 20$

   d. From $n = 15$ to $n = 18$

   e. From $n = 20$ to $n = 24$

2. The graph shows the total distance, in feet, walked by a person as a function of time, in seconds.



   a. Was the person walking faster between 20 and 40 seconds or between 80 and 100 seconds?

   b. Was the person walking faster between 0 and 40 seconds or between 40 and 100 seconds?

3. The height, in feet, of a squirrel running up and down a tree is a function of time, in seconds.

   Here are statements describing the squirrel's movement during four intervals of time. Match each description with a statement about the average rate of change of the function for that interval.

a. The squirrel runs up the tree very fast. b. The squirrel starts and ends at the same height. c. The squirrel runs down the tree. d. The squirrel runs up the tree slowly.	i. The average rate of change is negative. ii. The average rate of change is zero. iii. The average rate of change is small and positive. iv. The average rate of change is large and positive.

4. The percent of voters between the ages of 18 and 29 who participated in each United States presidential election between the years 1988 to 2016 are shown in the table.

Year	1988	1992	1996	2000	2004	2008	2012	2016
Percentage of voters ages 18–29	35.7	42.7	33.1	34.5	45.0	48.4	40.9	43.4

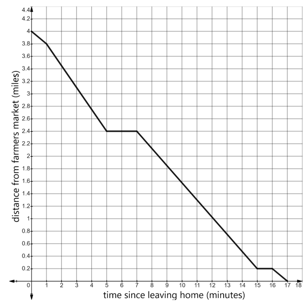
   The function P gives the percent of voters between 18 and 29 years old who participated in the election in year t.

   a. Determine the average rate of change for P between 1992 and 2000.

   b. Pick two different values of t so that the function has a negative average rate of change between the two values. Determine the average rate of change.

   c. Pick two values of t so that the function has a positive average rate of change between the two values. Determine the average rate of change.

5. Noah and a sibling are going to make their favorite dinner. To find the ingredients, they take a Lyft ride from their home to Rosa Parks Farmers Market. This graph represents function d, which gives his distance from the farmers market, in miles, m minutes since leaving his home.



   a. Estimate the average rate of change over Noah's entire route. Interpret this quantity for the context of the situation.

   b. What is the average rate of change from $m = 5$ to $m = 7$ and from $m = 15$ to $m = 16$? Interpret this quantity for the context of the situation and provide a possible explanation for this average rate of change.

6. Jada walks to school. The function D gives her distance from school, in meters, as a function of time, in minutes, since she left home.

   What does $D(10) = 0$ represent in this situation?

   (From Unit 5, Lesson 2)

7. Jada walks to school. The function D gives her distance from school, in meters, t minutes since she left home.

   Which equation tells us, "Jada is 600 meters from school after 5 minutes"?

   a. $D(5) = 600$
   b. $D(600) = 5$
   c. $t(5) = 600$
   d. $t(600) = 5$

   (From Unit 5, Lesson 2)

8. A news website shows a scatter plot with a positive relationship between the number of vending machines in a school and the average percentage of students who are absent from school each day. The headline reads, "Vending machines are causing our youth to miss school!"

    a. What is wrong with this claim?

    b. What is a better headline for this information?

    (From Unit 4)

9. Select **all** numbers that are solutions to the inequality: $-\frac{1}{2}x - 8 < 4x + 5\frac{1}{2}$

    a. 5
    b. -5
    c. 3
    d. -3
    e. 0

    (From Unit 2)

10. An airplane needs to begin its descent to land at Charlotte Douglas International Airport. The equation $h = 27000 - 1800m$ represents the height, h, in feet of the airplane m minutes after beginning its descent.

    Identify the slope, horizontal intercept, and vertical intercept of the graph of this equation, and explain their meaning in the context of the airplane's descent.

    (Addressing NC.8.F.4)

Lesson 8: Interpreting and Creating Graphs

PREPARATION

Lesson Goals	Learning Targets
• Given a verbal or visual representation of a situation, sketch a graph and show key features. • Interpret the average rate of change in a situation. • Practice interpreting key features of graphs and explaining (orally and in writing) their meaning in terms of a situation.	• When given a description or a visual representation of a situation, I can sketch a graph that shows important features of the situation. • I can explain the average rate of change of a function in terms of a situation. • I can make sense of important features of a graph and explain what they mean in a situation.

Lesson Narrative

By now, students have had multiple opportunities to interpret graphs of functions and to create them (primarily by plotting known input-output pairs of a function or by using descriptions of the situation). Students have also acquired essential vocabulary to communicate about graphs of functions, and used average rate of change as a way to measure how a function changes.

In this lesson, students apply these insights and skills to interpret or create graphs of functions that are less well defined and that model real-life situations that are more complex. The lesson includes two main activities about flag-raising and two optional activities that use other contexts.

Information about the functions is presented in the form of verbal descriptions, video clips, and images. More ambiguity is involved here than in cases students have previously encountered, so they will need to persevere in sense making and problem solving (MP1). At times, the information given may be inadequate, so students will need to make assumptions and decisions in order to produce graphs that show the desired behaviors or meet certain requirements. Along the way, students engage in important aspects of mathematical modeling (MP4).

In what ways might this lesson give students opportunities to surprise you with their thinking or reasoning?

Focus and Coherence

Building On	Addressing
NC.8.F.4: Analyze functions that model linear relationships. ● Understand that a linear relationship can be generalized by $y = mx + b$. ● Write an equation in slope-intercept form to model a linear relationship by determining the rate of change and the initial value, given at least two (x, y) values or a graph. ● Construct a graph of a linear relationship given an equation in slope-intercept form. ● Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of the slope and y-intercept of its graph or a table of values.	**NC.M1.F-IF.4:** Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; and maximums and minimums. **NC.M1.F-IF.6:** Calculate and interpret the average rate of change over a specified interval for a function presented numerically, graphically, and/or symbolically.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*15 minutes*)
  - Devices are required for the digital version of the extension in the first main activity, Flag Raising (Part One).
- **Activity 2** (*10 minutes*)
  - Prepare access to the video clip needed in Activity 2, Flag Raising (Part Two): https://bit.ly/RaisingAFlag
- **Activity 3** (*Optional, 20 minutes*)
- **Activity 4** (*Optional, 15 minutes*)
  - Prepare access to the video clip needed in Activity 4, The Bouncing Ball:
    - Tennis Ball Drop Full Speed: https://bit.ly/BouncingBallFullSpeed
    - Tennis Ball Drop Half Speed: https://bit.ly/BouncingBallHalfSpeed
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U5.L8 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional*, *5 minutes*)

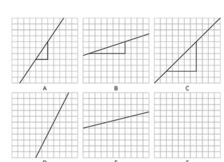
Building Towards: NC.8.F.4

In this bridge, students review how to identify given lines with different slopes and draw a line with a particular slope. In grade 8, students learned to draw slope triangles and divide the vertical change by the horizontal change in two points on the line to calculate the slope. Encourage these strategies and validate others in matching the slopes.

Student Task Statement

Here are several lines:[1]

1. Match each line shown with a slope from this list: $\frac{1}{3}$, 2, 1, 0.25, $\frac{1}{2}$, $\frac{3}{2}$
2. One of the given slopes does not have a line to match. Draw a line with this slope on the empty grid (F).



[1] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

DO THE MATH	PLANNING NOTES

Warm-up: Temperature Over Time (*5 minutes*)

Instructional Routine: Which One Doesn't Belong?
Addressing: NC.M1.F-IF.4

This warm-up prompts students to carefully analyze and compare the properties of four graphs in order to name *Which One Doesn't Belong*. Each graph represents a function relating time and temperature. No scales are shown on the coordinate axes, so students need to look for and make use of the structure of the graphs in determining how each one is like or unlike the others (MP7).

In making comparisons, students have a reason to use language precisely (MP6), especially recently learned terms that describe features of graphs.

WHICH ONE DOESN'T BELONG?	***What Is This Routine?*** Students are presented with four figures, diagrams, graphs, or expressions with the prompt: "Which one doesn't belong?" Typically, each of the four options "doesn't belong" for a different reason, and the similarities and differences are mathematically significant. Students are prompted to explain their rationale for deciding that one option doesn't belong and given opportunities to make their rationale more precise. ***Why This Routine?*** *Which One Doesn't Belong?* fosters a need to define terms carefully and use words precisely (MP6) in order to compare and contrast a group of mathematical objects or representations. Because there are no wrong answers, the focus is on student reasoning, and especially on students communicating their reasoning. This routine cultivates an inclusive classroom culture by prompting students to be creative thinkers, clear communicators, and good listeners.

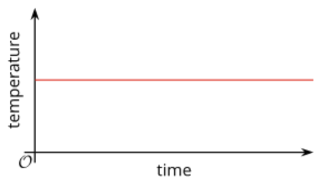
Step 1

- Ask students to arrange themselves in small groups or use visibly random grouping.
- Display the graphs for all to see.
- Give students 1 minute of quiet think time and then time to share their thinking with their small group. In their small groups, ask each student to share their reasoning as to why a particular item does not belong, and together, try to find at least one reason each item doesn't belong.

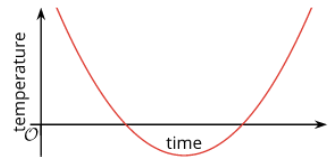
Student Task Statement

Which graph doesn't belong? Explain your reasoning.

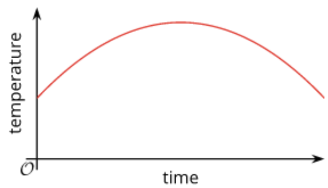
a.



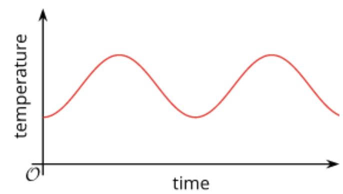
b.



c.



d.



Step 2

- Ask each group to share a reason why one of the items does not belong. Record and display the responses for all to see. Encourage students to use relevant mathematical vocabulary in their explanations, and ask students to explain the meaning of any terminology that they do use, such as "intercepts," "minimum," or "linear functions."
- After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question asking which one does not belong, attend to students' explanations and ensure the reasons given are correct. Press students on unsubstantiated claims. For example, if they claim that the vertical intercept of graph C is different than all the others, ask them how they know (as none of the vertical axes have a scale).

DO THE MATH

PLANNING NOTES

Activity 1: Flag Raising (Part One) (*15 minutes*)

Instructional Routine: Take Turns
Addressing: NC.M1.F-IF.4

Prior to this point, students have had multiple opportunities to interpret graphs of functions in terms of a situation. In most cases, students were given some information about the units of measurement of at least one quantity, which helped them reason about the functions being represented.

In this activity, students encounter graphs without units on either of the axes. They interpret the graphs in the context of raising a flag and make a case for whether the graphs are realistic (which may depend on the units chosen for the axes). Students also see for the first time a graph that is a vertical line and think about why this graph does not represent a function.

As students relate features of graphs to behaviors of quantities in a situation, they reason quantitatively and abstractly (MP2). In explaining and discussing why a graph is or is not realistic, students practice constructing logical arguments and critiquing the reasoning of others (MP3).

Step 1

- Show pictures of a flag ceremony and describe flag raising to students who might be unfamiliar with it. Consider sharing that, in some countries, students hold a flag-raising ceremony on a daily or weekly basis.
- Ask students to arrange themselves in pairs or use visibly random grouping.
- Ask students to read the task statement and look at graphs. Then, give students a moment of quiet time to think about the following questions:
  - "What might be reasonable units to use for the axes of the graphs?"
  - "How do we know that each graph represents a function?"

RESPONSIVE STRATEGIES

Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their strategy. For example, "I notice that. . .", "I wonder . . .", "That could/couldn't be true because . . ."

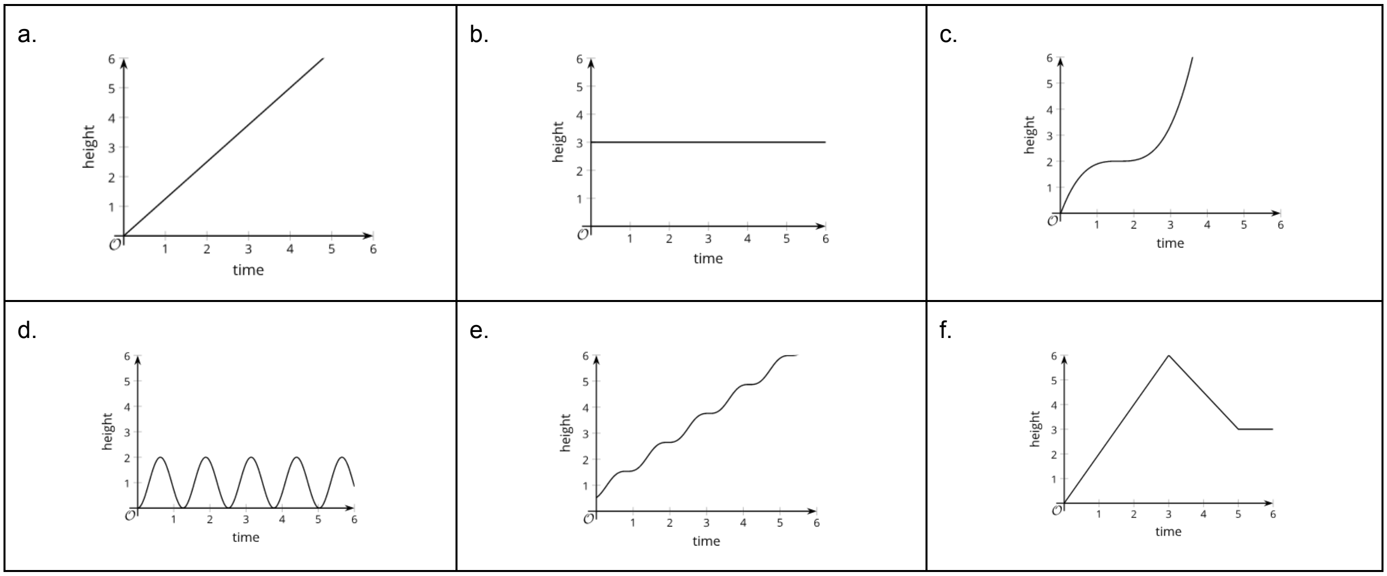
Supports accessibility for: Language; Social-emotional skills

- Give students a moment to share their thinking with their partner. Follow with a brief whole-class discussion.
  - Students are likely to suggest seconds as a reasonable unit to use for time and meters, feet, or yards as a reasonable unit for height. If these units are not mentioned, ask students about them.
- Ask partners to *Take Turns* interpreting the graphs in the first question. Students may not have time to interpret every graph in question 1. To ensure that all graphs have equal representation in the whole-group discussion, consider assigning specific orders to each pair of students. For example, the first student pair could complete the graphs in the order a, b, c, d, e, f, but the second student pair may be assigned the order b, c, d, e, f, a, and a third pair of students may be assigned the order c, d, e, f, a, b, and so on.
- Ask students to pause for a discussion after about 3 minutes of taking turns before they move to the second question.

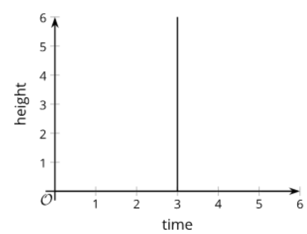
Student Task Statement

A flag ceremony is held at a Fourth of July event. The height of the flag is a function of time.

Here are some graphs that could each be a possible representation of the function.



1.
    a. For each graph assigned to you, explain what it tells us about the flag.
    b. Decide as a group which graph(s) appear to be most realistic and which ones are least realistic.
2. Here is another graph that relates time and height.



    a. Can this graph represent the time and height of the flag? Explain your reasoning.
    b. Is this a graph of a function? Explain your reasoning.

Are You Ready For More?

Suppose an ant is moving at a rate of 1 millimeter per second and keeps going at that rate for a long time.

If time, x, is measured in seconds, then the distance the ant has traveled in millimeters, y, is $y = 1x$. If time, x, is measured in minutes, the distance in millimeters is $y = 60x$.

1. Explain why the equation $y = (365 \cdot 24 \cdot 3,600)x$ gives the distance the ant has traveled, in millimeters, as a function of time, x, in years.
2. Use graphing technology to graph the equation.
    a. Label the axes with appropriate quantities and units.
    b. Does the graph look like that of a function? Why do you think it looks this way?
3. Adjust the graphing window until the graph no longer looks this way. If you manage to do so, describe the graphing window that you use.
4. Do you think the last graph in the flag activity could represent a function relating time and height of flag? Explain your reasoning.

Step 2

Ask for volunteers to explain what each graph means in the context of flag raising and give an argument as to why it is or is not realistic.

- Discuss with students which graph is the most realistic and why. Also discuss whether using certain units of measurements would make an unrealistic graph more realistic, or vice versa. (For example, none of the graphs would be realistic if the time is measured in days or months and the height in kilometers or centimeters.)
- Next, solicit some explanations as to why the vertical line does not represent a function. If not mentioned in students' explanations, bring up a key point: a vertical line can be interpreted to mean that for one particular input ($t = 3$, in this case), there is an infinite number of possible outputs; therefore, it cannot represent a function.

DO THE MATH	PLANNING NOTES

Activity 2: Flag Raising (Part Two) (*10 minutes*)

Instructional Routine: Compare and Connect (MLR7)
Addressing: NC.M1.F-IF.4; NC.M1.F-IF.6

Previously, students analyzed several graphs to see if they could represent the height of a flag being raised in a ceremony. In this activity, students watch a video of a flag being raised. They sketch a possible graph to represent the height of the flag as a function of time and then use their graphs to estimate the rate of change of the flag.

Students' graphs do not need to be very precise, but they should capture key features of the situation. To do so, students need to make some estimates. For example, they need to gauge the starting height of the flag, the height of the pole, the heights at which pauses happened, whether the flag was moving at a constant rate between the pauses, and so on. Students also need to make some decisions about the graph (for example, the scale to use on each axis, whether the graph should be discrete or continuous, and so on). Along the way, students engage in aspects of modeling (MP4).

If desired and if possible, provide each student or each group with access to the video so students can replay the clip a few times, as needed. As students work, identify those who make different decisions for their graph and can explain their rationale. Ask them to share their work later.

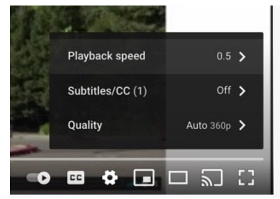
If time is limited, focus work time and discussion on the graphing portion (the first question).

Step 1

- Tell students that they will watch a video of a flag being raised, and their job is to sketch a graph that represents the height of the flag, in feet, as a function of time, in seconds. Students may wish to see the video a few times to help them sketch a graph.

- Before playing the video, ask students to think about what information or quantities to look for while watching the video. If possible, record and display their ideas for all to see.
- Explain to students that their graphs do not need to be precise and some estimations are required, but the graphs should reasonably capture the movement of the flag. Video “Raising a Flag (Full Speed)” available here: https://bit.ly/RaisingAFlag

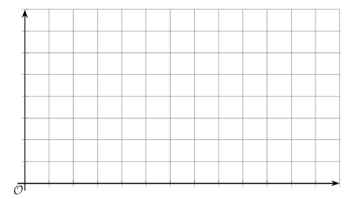
Advancing Student Thinking: Some students may have trouble starting their graphs because they don’t know what upper limits to use for the axes. Ask them to watch the video clip again and try to gather some information that may help them decide on the upper limits. Assure them that some estimation and decision making are necessary. Some students may benefit from watching the video clip at half-speed. If viewing in Google Chrome, you can adjust the playback speed under settings (gear icon). The flag begins lowering at exactly 13 seconds into the video. That is available here: https://bit.ly/RaisingAFlag.



Student Task Statement

Your teacher will show a video of a flag being raised. Function H gives the height of the flag over time. Height is measured in feet. Time is measured in seconds since the flag is fully secured to the string, which is when the video clip begins.

1. On the coordinate plane, sketch a graph that could represent function H. Be sure to include a label and a scale for each axis.
2. Use your graph to estimate the average rate of change from the time the flag starts moving to the time it stops. Be prepared to explain what the average rate of change tells us about the flag.



Step 2

- Use the *Compare and Connect* routine to prepare students for the whole-class discussion. Invite students to quietly circulate and analyze at least two other graphs in the room.
  - Give students quiet think time to consider which features of the graphs are alike and which are different.
  - Display these prompts as students move around the room: “Where does the maximum height occur?”; “What scale is used on the axes?”; and “Where are the constant time intervals?”
  - Next, ask students to find a partner to discuss what they noticed. This will help students compare and contrast key features on graphs of the same function.
- Select previously identified students to share their graph and explain their drawing decisions and briefly discuss:
  - how the graphs are alike and how they are different
  - key features such as intercepts, maximum, minimum, and intervals when the function increases, remains constant, or decreases
- If time permits, invite a couple of students to share how they used their graph to estimate the flag’s average rate of change. Emphasize that the average rate of change (which would vary for different graphs) represents the average amount of height the flag gained, per second, from the time it started being raised until the time it reached the top of the pole.

DO THE MATH	PLANNING NOTES

Activity 3: Two Pools (*Optional, 20 minutes*)

Instructional Routines: Aspects of Mathematical Modeling; Discussion Supports (MLR8)
Addressing: NC.M1.F-IF.4; NC.M1.F-IF.6

This optional activity that incorporates *Aspects of Mathematical Modeling* allows students to practice creating graphs of functions based on verbal descriptions. Students sketch graphs that could represent the heights of water in two pools, each as a function of time since the pools start to be filled.

The modeling demand in this activity is higher than in previous activities, as students are given less quantitative information and therefore need to make more assumptions and decisions (MP4). For example, they need to consider whether and how the shape and size of each pool affect the rate at which the water rises, how much longer it would take to fill the large pool compared to the small pool, how moving a hose from one pool to the other changes the graph of each function, and so on.

Students are not expected to draw precise graphs. What is more important is that the graphs of the two functions are comparatively correct and reasonable in terms of the situation, that students consider the quantities carefully, and that they can explain their decisions.

As students work, look for different assumptions and decisions students make that result in a variety of graphs. Consider asking them to share their graphs and reasoning during the whole-class discussion.

ASPECTS OF MATHEMATICAL MODELING	***What Is This Routine?*** In activities tagged with this routine, students engage in scaled-back modeling scenarios, for which students only need to engage in a part of a full modeling cycle. For example, they may be selecting quantities of interest in a situation or choosing a model from a list. ***Why This Routine?*** Mathematical modeling is often new territory for both students and teachers. Activities tagged as *Aspects of Mathematical Modeling* offer opportunities to develop discrete skills in the supported environment of a classroom lesson to make success more likely when students engage in more open-ended modeling.

Step 1

- Keep students in pairs. Ask students to read the activity statement and be prepared to ask any clarifying questions about the task.
- After answering students' questions, give students a few minutes of quiet time to sketch the graphs for the first situation and then time to discuss the graphs with their partner. Tell partners to discuss their assumptions about the situation and the reasonableness of the graphs based on those assumptions. Ask them to revise their graphs based on each other's feedback.

RESPONSIVE STRATEGY

Use color coding and annotations to highlight connections between representations in a problem. For example, ask students to graph the function for the large pool in one color and the function for the small pool in another color and highlight relevant information in the description using the same colors.

Supports accessibility for: Visual-spatial processing

- Consider selecting a few students to share their graphs and graphing decisions before the class continues to the second and third situations. Discuss questions such as:
  - "The water in both hoses flows at a constant rate. Did you assume the constant rate in one hose to be the same as the constant rate in the other hose?"
  - "Did you assume that the water for the two hoses was turned on at the same time?"
  - "What assumptions did you make about the shape and size of each pool? Did you assume the pools to have the same area for their base and just have different heights, or that they have different base areas?"

Advancing Student Thinking: Some students might mistakenly think that when the pools are "full," the water in each pool has reached the same height. Remind students that the two pools have different heights, so it takes different heights of water to make them full.

Student Task Statement

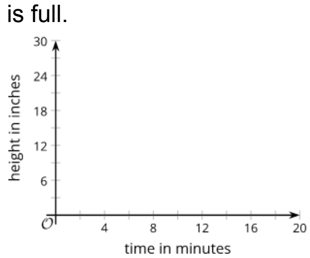
To prepare for a backyard party, a parent uses two identical hoses to fill a small pool that is 15 inches deep and a large pool that is 27 inches deep.

The height of the water in each pool is a function of time since the water is turned on.

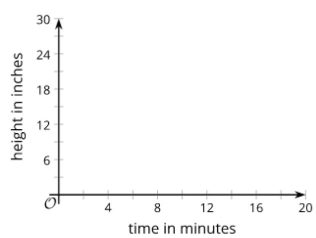
Here are descriptions of three situations. For each situation, sketch the graphs of the two functions on the same coordinate plane, so that $S(t)$ is the height of the water in the small pool after t minutes, and $L(t)$ is the height of the water in the large pool after t minutes.

In both functions, the height of the water is measured in inches.

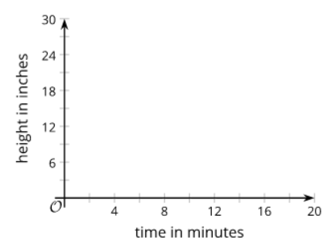
Situation 1: Each hose fills one pool at a constant rate. When the small pool is full, the water for that hose is shut off. The other hose keeps filling the larger pool until it is full.



Situation 2: Each hose fills one pool at a constant rate. When the small pool is full, both hoses are shut off.



Situation 3: Each hose fills one pool at a constant rate. When the small pool is full, both hoses are used to fill the large pool until it is full.



Step 2

- Invite or select students to share their graphs and the assumptions and decisions they made as they were graphing. Display graphs that correctly represent the situation but look different because of variation in assumptions.

Use *Discussion Supports* to support whole-class discussion. For each graph that is shared, ask students to use precise mathematical language to restate how the author's assumptions and decisions affected key features of the graph. Ask the original speaker if their peer was able to accurately restate their thinking. Call students' attention to any words or phrases that helped to clarify the original statement. This routine provides more students with an opportunity to produce language as they interpret the reasoning of others.

- Discuss questions such as:
  - "How would the vertical values of the two graphs compare when the pools are full?" (The vertical value of the large pool would be up to 12 inches greater than that of the small pool because the large pool is 12 inches taller than the small pool.)
  - "When each pool is being filled by one hose and at the same rate, should the two graphs have the same slope? Why or why not? If not, which graph has a greater slope?" (No. The water in the smaller pool rises more quickly because the area of the pool's base is smaller, so the slope for that graph should be greater.)
  - "How would the graph for the large pool change when one hose was moved from the small pool to the large pool? Would its slope increase, decrease, or stay the same?" (The slope of the graph for the large pool would double because water is now rising at twice its previous rate. The graph for the small pool would stay constant, as the water is no longer increasing in height.)

DO THE MATH	PLANNING NOTES

Activity 4: The Bouncing Ball (*Optional, 15 minutes*)

Instructional Routine: Stronger and Clearer Each Time (MLR1) - Responsive Strategy	
Addressing: NC.M1.F-IF.4	

This optional activity gives students another opportunity to represent the quantities in a situation with a table and a graph, identify key features of the graph, and interpret those features in terms of the situation.

The Bouncing Ball will be used again in a subsequent lesson. Students do not need to have this activity completed to access the other Bouncing Ball activity.

Step 1

- Before students begin this activity, ask them to think about what information or quantities they should look for in the still images.
- The following are some videos that show the same clip of a ball being dropped, but played back at different speeds. Show one or more of the videos for all to see. Before showing the videos, ask students to think about what information or quantities to look for while watching the videos.
  - Tennis Ball Drop Full Speed: https://bit.ly/BouncingBallFullSpeed
  - Tennis Ball Drop Half Speed: https://bit.ly/BouncingBallHalfSpeed

RESPONSIVE STRATEGY

Use this routine to help students improve their verbal and written responses to the question, "Find the maximum and minimum values of the function. Explain what they tell us about the tennis ball." Give students time to meet with 2–3 partners to share and get feedback on their responses. The listener should press for detail and precision in language by asking: "Can you say that a different way?"; "How do you know?"; or "Can you show this on the table, graph, and screen shots?" Invite students to go back and revise or refine their written explanation based on the feedback from their peers. This will help students uncover any gaps in their interpretation of a graph, fine-tune their understanding of what key graphical features represent, and express their interpretation more clearly and precisely.

Stronger and Clearer Each Time (MLR1)

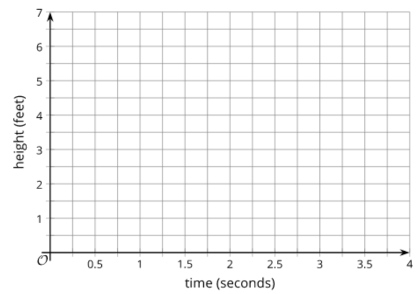
Advancing Student Thinking**:** Some students may mistake the horizontal axis on the graph to represent horizontal distance rather than time. Because the movement of the bouncing ball is primarily up and down (except toward the end, when it began rolling), these students might sketch a graph that is essentially a series of overlapping vertical segments. Ask them to revisit the input variable and what the horizontal axis represents. Then, ask them to plot some points for different values of t.

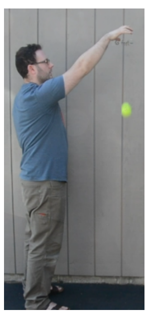
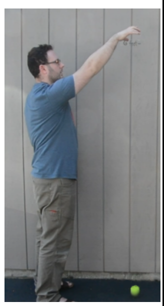
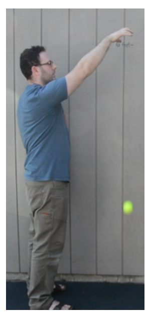
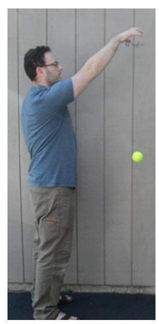
Student Task Statement

Here are some still images of a tennis ball being dropped. The height of the ball is a function of time. Suppose the height is h feet, t seconds after the ball is dropped.

1. To help you get started, here are some pictures and a table. Complete the table with your estimates before sketching your graph.

Time (seconds)	Height (feet)
0	
0.28	
0.54	
0.74	
1.03	
1.48	
1.88	
2.25	



0 seconds	0.28 seconds	0.54 seconds	0.74 seconds	1.03 seconds	1.48 seconds	1.88 seconds	2.25 seconds
							

2. Use the blank coordinate plane to sketch a graph of the height of the tennis ball as a function of time. Assume that the ball reaches a high point at 0, 1.03, and 1.88 seconds.

3. Identify horizontal and vertical intercepts of the graph. Explain what the coordinates tell us about the tennis ball.

4. Find the maximum and minimum values of the function. Explain what they tell us about the tennis ball.

Step 2

- Discuss how students translated the movement of the dropped ball into a graph and on the connections between the graph and the situation. Ask questions such as:
  - "In this case, the horizontal intercepts are also the minimums of the graph. Why is that?" (The ball could not have a height that is less than 0. The lowest point it could be is 0 feet.)
  - "What happened to the output value, h, as the input value, t, increased?" (Overall, the output decreased as t increased, but there were intervals—whenever the ball bounced up—in which the height of the ball increased as time increased.)
- If time permits, discuss students' responses to the last two questions.

DO THE MATH

PLANNING NOTES

Lesson Debrief (*5 minutes*)

The purpose of this lesson is for students to be able to tell stories visually with a graph by connecting key ideas such as rate of change, horizontal and vertical intercepts, and intervals of increase and decrease to the context of the story.

Choose whether students should first have an opportunity to reflect in their workbooks or brainstorm with a partner before sharing their observations with the class.

Representing a function with a graph requires careful attention to the behavior of the quantities in the situation. To reinforce this idea, tell students that they will observe you walking from one end of the classroom to the other, and suppose they were to graph your distance from the starting point as a function of time.

Ask students to notice any characteristics of the movement that would affect the output of this function and show up as distinct features on a graph.

Walk from one side of the classroom to the other, incorporating moves such as:

- walking at a constant rate
- slowing down and speeding up
- walking one way and then turning around and walking back
- taking a few steps, stopping and standing still, and then continuing walking
- walking backward a few steps, then walking forward again

Solicit as many observations as possible about the walk. For each observation that students make, ask them how it would affect a graph of the function. For instance, students may note that slowing down and speeding up translate to shallower and steeper lines, respectively. They may also say that stopping translates to a horizontal segment on the graph, and that the longer the pause, the longer the segment.

If time permits, repeat the walking demonstration and ask students to graph the function.

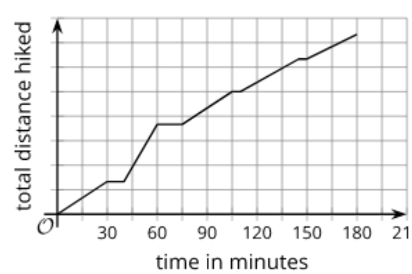
PLANNING NOTES

Student Lesson Summary and Glossary

We can use graphs to help visualize the relationship between quantities in a situation, even if we only have a general description.

Here is a description of a hiker's journey on a trail:

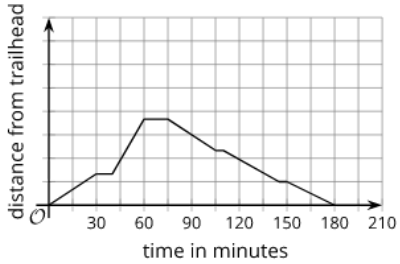
A hiker walked briskly and steadily for about 30 minutes and then took a 10-minute break. Afterward, she jogged all the way to the end of the trail, which took about 20 minutes. There, she took a 15-minute break, and then started a leisurely walk back, stopping twice to enjoy the scenery. Her return trip along the same trail took 105 minutes.



We can sketch a graph of the distance the hiker has traveled as a function of time based on this description.

Even though we don't know the specific distances she has traveled or the length of the trail, the graph can show some important features of the situation. For example:

- the intervals in which the distance increased or stayed constant
- when the distance was increasing more quickly or more slowly
- the time when the total distance hiked was greatest.



If we are looking at distance from the trailhead (the start of the trail) as a function of time, the graph of the function might look something like this:

It shows the distance increasing as the hiker was walking away from the trailhead, and then decreasing as she was returning to the trailhead.

Cool-down: Caught in a Tree (*5 minutes*)

Addressing: NC.M1.F-IF.4

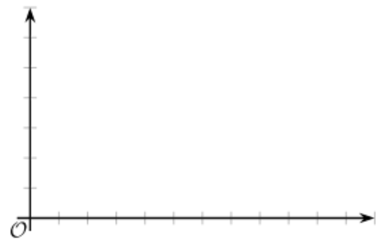
Cool-down Guidance: Points to Emphasize
If students struggle to start the graph because there are no numbers, consider taking some time in a subsequent lesson to model a think-aloud for doing this cool-down, and then having students try practice problem 3 from Lesson 8. Students will revisit the concept of y-intercept as the "starting point" of a graph, x-intercept as "ground-level" in Units 6 and 7, so they will also have more chances to practice these ideas.

Cool-down

A child tosses a baseball up into the air. On its way down, it gets caught in a tree for several seconds before falling down to the ground.

Sketch a graph that represents the height of the ball, h, as a function of time, t.

Be sure to include a label and a scale for each axis.



Student Reflection:

What helps you learn best in class? What can be done differently to help you learn your best?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

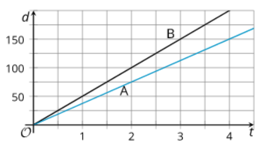
What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

Students shared their thinking multiple times in this lesson. What have you noticed about the language students use? What support can you offer to students who struggle to communicate their ideas orally?

Practice Problems

1. The graphs show the distance, d, traveled by two cars, A and B, over time, t. Distance is measured in miles and time is measured in hours.

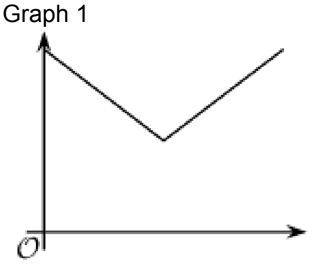
   Which car traveled slower? Explain how you know.



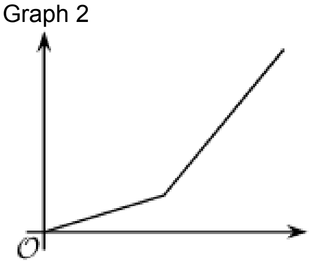
2. Here are descriptions of four situations in which the volume of water in a tank is a function of time. Match each description to a corresponding graph.

   a. An empty 20-gallon water tank is filled at a constant rate for 3 minutes until it is half full. Then, it is emptied at a constant rate for 3 minutes.
   b. A full 10-gallon water tank is drained for 30 seconds, until it is half full. Afterwards, it gets refilled.
   c. A 2,000-gallon water tank starts out empty. It is being filled for 5 hours, slowly at first, and faster later.
   d. An empty 100-gallon water tank is filled in 50 minutes. Then, a dog jumps in and splashes around for 10 minutes, letting 7 gallons of water out. The tank is refilled afterwards.

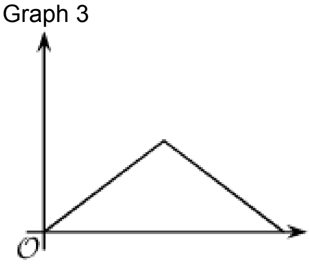
Graph 1



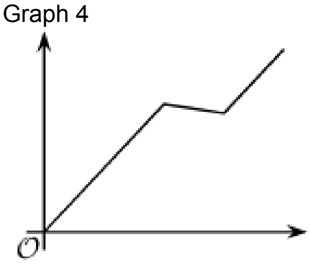
Graph 2



Graph 3



Graph 4



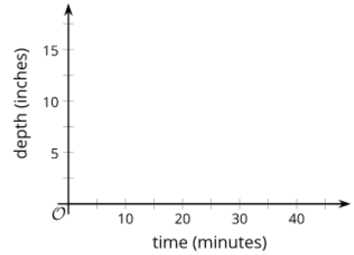
3. Clare describes her morning at school yesterday: “I entered the school on the first floor, then walked up to the third floor and stayed for my class for an hour. Afterwards, I had an hour-long class in the basement, and after that I went back to the ground level and sat outside to eat my lunch."

   Sketch a possible graph of her height from the ground floor as a function of time.

4. Tyler filled up their bathtub, took a bath, and then drained the tub. The function gives the depth of the water, in inches, t minutes after they began to fill the bathtub.

   These statements describe how the water level in the tub was changing over time. Use the statements to sketch an approximate graph of function.

   - $B(0) = 0$
   - $B(1) < B(7)$
   - $B(5) - B(0) = 6$
   - $B(9) = 11$
   - $B(10) = B(23)$
   - $B(20) > B(40)$

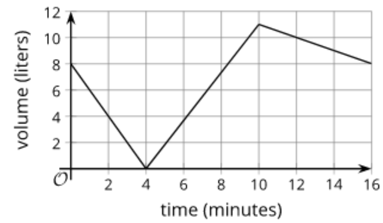


5. Function V gives the volume of water (liters) in a water cooler as a function of time, t (minutes). This graph represents function V.

   a. What is the greatest water volume in the cooler?

   b. Find the value or values of t that make $V(t) = 4$ true. Explain what the value or values tell us about the volume of the water in the cooler.

   c. Identify the horizontal intercept of the graph. What does it tell you about the situation?



   (From Unit 5, Lesson 6)

6. Two functions are defined by these equations:

   $$f(x) = 5.1 + 0.8x$$
   $$g(x) = 3.4 + 1.2x$$

   Which function has a greater value when x is 3.9? How much greater?

   (From Unit 5, Lesson 5)

7. Function f is defined by the equation $f(x) = 3x - 7$. Find the value of c so that $f(c) = 20$ is true.

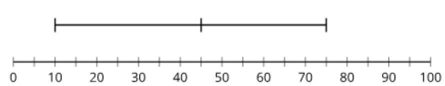
   (From Unit 5, Lesson 5)

8. B is the midpoint of segment AC on the coordinate plane. The coordinates of the 3 points are: $A(-4, 5)$; $B(x, 2)$; and $A(-4, 5)$; $B(x, 2)$; and $C(9, y)$. What are the values of x and y?

   (From Unit 3)

9. Noah draws this box plot for a data set that has an IQR of 0.

   Explain why the box plot is complete even though there do not appear to be any boxes.



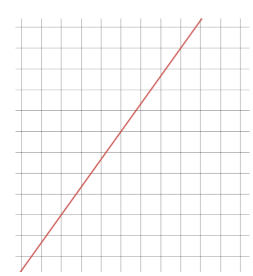
   (From Unit 1)

10. For the line shown below:

    a. What is the slope of the line?

    b. Draw a line with a greater slope. What is the slope of your new line?

    c. Draw a line with a smaller positive slope. What is the slope of your new line?



    (Addressing NC.8.F.4)

Lessons 9 & 10: Checkpoint

PREPARATION

Lesson Goals	Learning Targets
• Learn and grow mathematically in course-level content. • Communicate and address mathematical areas of strength and areas of growth.	• I can continue to grow as a mathematician and challenge myself. • I can share what I know mathematically.

Lesson Narrative

This is a Checkpoint day. These lessons have three main purposes: 1. differentiated and small-group instruction; 2. opportunities for students to participate in various learning stations to refine and extend previous learning; and 3. the opportunity for students to complete the next unit's Check Your Readiness (CYR). Administering the CYR at this point in the unit allows plenty of time for the data to inform the next unit's instruction. Checkpoint days consist of two lessons (one full block) and are structured as four 20-minute stations that students rotate between. There are a total of seven stations students can engage with. Since students will not be able to participate in all seven stations, please note that Station A (Unit 6 Check Your Readiness) is required for all students.

This Checkpoint does not include an Are You Ready For More? Station, as many of these additional opportunities are not stand alone activities but connect back to the activities in the given lesson. If students did not previously complete these Are You Ready For More? activities, an additional station option would be for students to do so today.

A. Unit 6 Check Your Readiness (*Required*)
B. Teacher-led Small-group Instruction
C. Graphs and Situations
D. Points into Function Notation and Back
E. Food Insecurity in Mecklenburg County
F. Micro-Modeling
G. Practice with Rational Bases

How will you determine which stations individual students participate in? How can you organize the stations in a way that empowers students' development of mathematical identities, provides essential support, and limits the appearance of "smart" or "not-smart" assignments?

Agenda, Materials, and Preparation

- **Station A** (*Required, 20 minutes*)
  - Unit 6 Check Your Readiness (print 1 copy per student)
- **Station B** (*20 minutes*)
- **Station C** (*20 minutes*)
  - Student access to technology to watch videos from http://www.graphingstories.com/
- **Station D** (*20 minutes*)
  - Graph paper (1 sheet per student)
- **Station E** (*20 minutes*)
- **Station F** (*20 minutes*)
- **Station G** (*20 minutes*)
- **Station H** (*20 minutes*)

STATIONS

Station A: Unit 6 Check Your Readiness (*Required, 20 minutes*)

Remind students that it is really important that their responses to these questions accurately represent what they know. Ask them to answer what they can to the best of their ability. If they get stuck, they should name what they don't know or understand.

Station B: Teacher-led Small-group Instruction (*20 minutes*)

Use student cool-down data, Check Your Readiness Unit 5 data, and informal formative assessment data from Unit 5 (Lessons 1–8) to provide targeted small-group instruction to students who demonstrate the need for further support or challenge on topics taught up to this point.

Potential topics:

- Defining a function
- Function notation
- Interpret key features of graphs, tables, and verbal descriptions in context to describe functions
- Average rate of change
- Creating graphs of functions

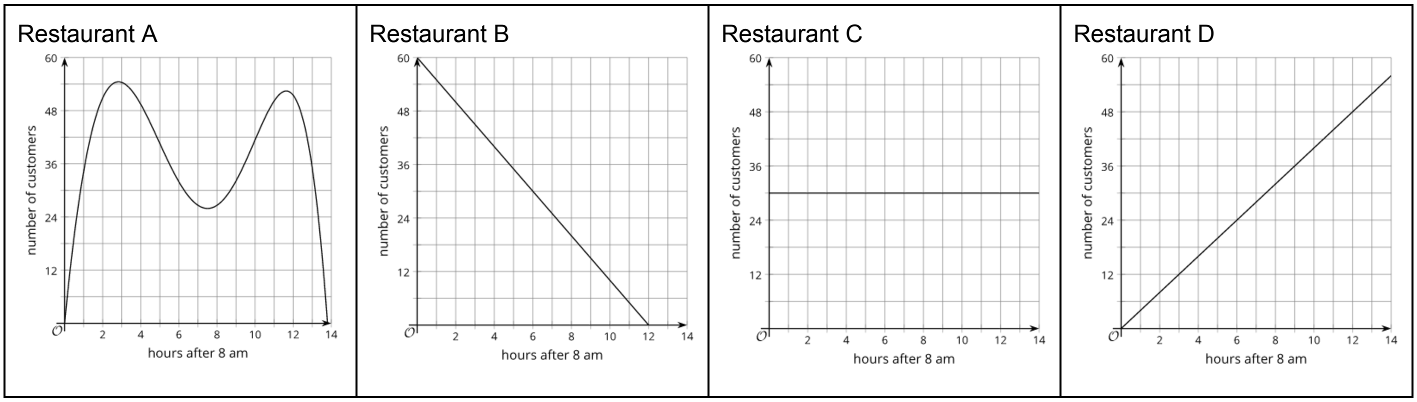
Station C: Graphs and Situations (*20 minutes*)

For question 1 in this station, students interpret graphs representing the number of customers at a restaurant as a function of time. Students are given scenarios and select the best restaurant for each scenario. Students review the graphs and some options for interpreting the graphs as it relates to the time of day in each restaurant in their Student Workbooks.

For question 2, students watch videos from the website (http://www.graphingstories.com/) and graph the situations in their Student Workbook. There are 21 videos. Allow students to choose which three they will analyze or determine in advance which videos students should engage with. This station supports students who may need extra practice with interpreting and creating graphs then relating them to situations.

Station C

1. These graphs show how busy restaurants are at different times of the day.



   a. Which restaurant is busy in the morning, then has fewer customers in the evening?

   b. If Lin's mom wants to go to a popular dinner restaurant, which restaurant should Lin take her mom to eat?

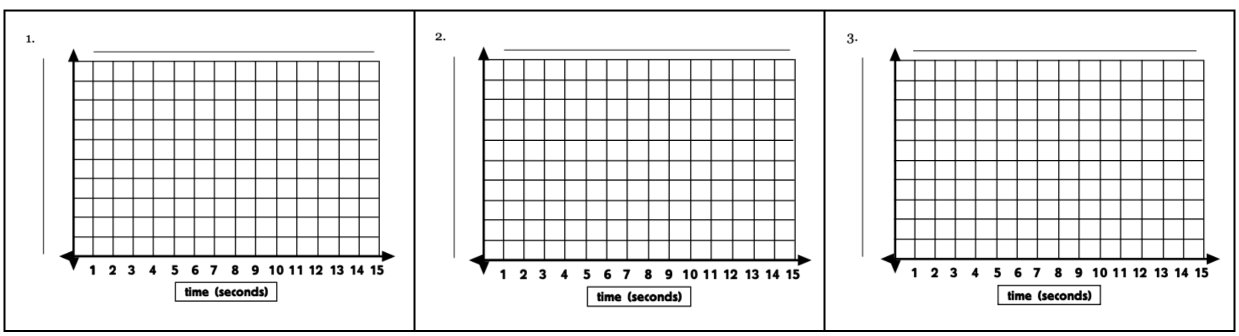
   c. Noah's dad prefers breakfast places with few customers so that he can start on work while eating. Which restaurant should Noah's dad go to for breakfast?

   d. Which restaurant would you visit during a 30-minute lunch break?

2. Your teacher will give you a link to some videos. Choose any three videos to watch or watch the videos your teacher has assigned and follow the steps below:[1]

   a. Watch a video, pausing after you watch the slow-motion version.

   b. Notice the timer in the lower left of the video. Use this timer and your best estimate of the heights to graph the motion in the video. Pause or rewind the video as often as you need to.



   c. Watch the video to the end and compare your graph with the one in the video.

   d. Repeat as time permits.

[1] Adapted from www.graphingstories.com

DO THE MATH	PLANNING NOTES

Station D: Points into Function Notation and Back (*20 minutes*)

In this station, students practice their understanding of function notation for points by converting given coordinate pairs into function notation and then writing coordinate pairs associated with values given in function notation. This station provides support for students who need more practice with interpreting equations like $f(2) = 5$.

Station D

1. A function is given by the equation $y = f(x)$. Write each of these coordinate pairs in function notation.

    a. $(2, 3)$

    b. $(-1, 4)$

    c. $(0, 3)$

    d. $(4, 0)$

    e. $(\frac{2}{3}, \frac{3}{4})$

2. A function is given by the equation $h(x) = 5x - 3$. Write the coordinate pair for the point associated with the given values in function notation.

    a. $h(3)$

    b. $h(-4)$

    c. $h(\frac{2}{5})$

3. Get ready to play this game:

    One person thinks of a rule for a function. The other person (or other people, working together) is the guesser. The guesser asks for the output of the function for certain inputs. ("What is $f(5)$?", for example.) After hearing the response to each question, the guesser can take a guess at the rule.

    Here are two sample games, played by Priya and Andre.

Priya thinks of the rule $f(x) = 2x + 1$.

Andre (the guesser): “What is $f(12)$?” (pronounced "f of 12")
Priya: “25.”
Andre: “Is the rule $f(x) = x^2$?” (pronounced "f of x equals x squared")
Priya: “No.”

Andre: “Okay, then - what is $f(3)$?”
Priya: “7.”
Andre, thinks to himself: *The rule could be* $f(x) = 3x - 2$ *because* $3 \cdot 3 - 2 = 7$*. That doesn't work for* 12*, though, because* $3 \cdot 12 - 2 = 34$*, not* 25*. I need something that works for both. A simple way to make an output of* 7 *from an input of* 3 *is to multiply* 3 *by* 2*, then add* 1*. Does that work for an input of* 12*? Yes:* $2 \cdot 12 + 1 = 25$*.*
Andre: “Is the rule $f(x) = 2x + 1$?”
Priya: “Yes!”

Since Andre guessed the rule, now they switch and Priya is the guesser.

Andre thinks of the rule $f(x) = x^2 - 4$.

Priya: “What is $f(4)$?”
Andre: “12.”
Priya: “Is the rule $f(x) = 3x$?”
Andre: “No.”

Priya: “What is $f(3)$?”
Andre: “5.”
Priya, thinks to herself: *If the graph of this function is a line, it has a slope of* 7*.*
Priya: “Is the rule $f(x) = 7x - 16$? That works for both of the guesses so far.”
Andre: “No.”

Priya: “What is $f(-4)$?”
Andre: “12.”
Priya, thinks to herself: *Since my previous guess was wrong, I know this graph can't be a line. Maybe it involves* x^2*, since* $f(4)$ *is the same as* $f(-4)$*.*
Priya: “Is the rule $f(x) = x^2 - 4$?”
Andre: “Yes!”

Now play the game with a partner or a group, with one person thinking of a rule and everyone else trying to guess it in as few turns as possible. The function rules can involve any mathematical operations you know. It may help the guesser(s) to plot the (input, output) coordinates on graph paper.

After each round, switch so that someone else makes up the function rule.

DO THE MATH

PLANNING NOTES

Station E: Food Insecurity in Mecklenburg County (*20 minutes*)

Instructional Routine: Notice and Wonder

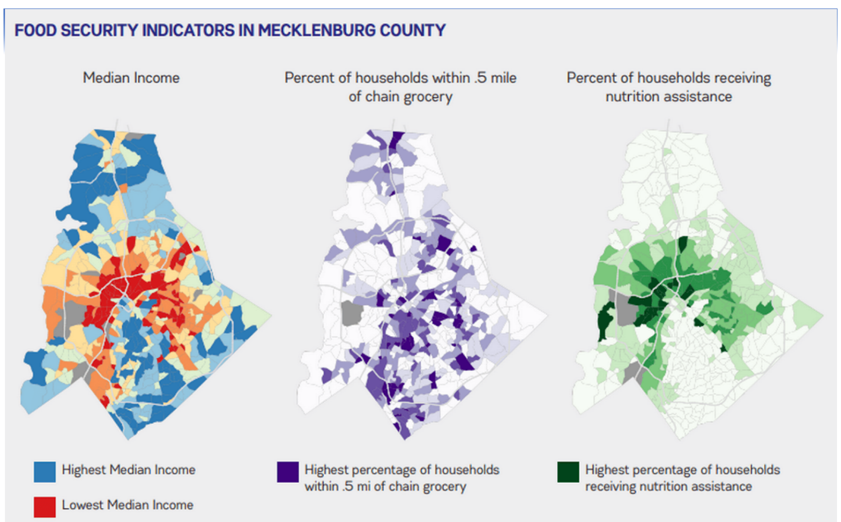
In this station, students apply their knowledge of linear models for bivariate data and interpreting function notation in context while revisiting the social phenomenon of food deserts and food insecurity. This station provides an opportunity for students to think critically about the mathematics related to an important human rights issue in their own community. In the beginning of the activity, students will use the *Notice and Wonder* routine.

Station E

Recall being introduced to "food deserts" in Unit 4, Lesson 1. The data explored in that lesson was for San Antonio, Texas. Now you will explore data related to food insecurity in Mecklenburg County.

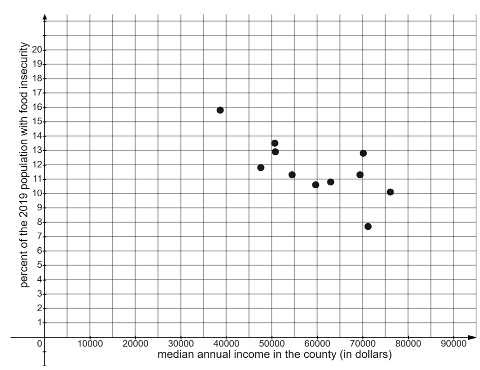
A household is considered to be food insecure when food intake of one or more household members is reduced and eating patterns are disrupted due to lack of money and resources. Food insecurity is defined as a lack of consistent access to enough food for every person in a household to live an active, healthy life.[2]

This graphic shows three factors that could be relevant to food insecurity. What do you notice? What do you wonder?



(Source[3])

One of the strategies Mecklenburg County has implemented to reduce food insecurity includes attempts to increase availability of fresh healthy food from farmers markets for families who qualify for government assistance for food. The data and scatter plot come from a study of counties around the country with varying levels of food insecurity in their populations.[4]



[2] Feeding America. *Food Insecurity.* https://www.feedingamerica.org/hunger-in-america/food-insecurity
[3] KarenKarp&Partners. (2017). *Unlocking the Potential of Charlotte's Food Systems and Farmers' Markets.* https://charlottenc.gov/HNS/CE/Documents/KKP_CharlotteFarmersMarketsFINAL.pdf
[4] Ibid.

Median annual income in the county	Percent of the 2019 population with food insecurity[5]
50685	13.5
71200	7.7
76097	10.1
70158	12.8
50815	12.9
59668	10.6
38681	15.8
54492	11.3
62978	10.8
69433	11.3
47607	11.8

The regression equation is given by the function $f(x) = -0.00013x + 19.36$, with x representing the median income and $f(x)$ representing the percentage of the population with food insecurity.

1. Interpret the slope and vertical intercept of the regression equation in terms of median income and percentage of the population with food insecurity.
2. The median annual income for Mecklenburg County when the data for this study was recorded was $62,978. Use the function that models this data, $f(x)$, to predict the percentage of food insecurity in 2019.
3. Compare the predicted value to the actual value, 10.8%. Is the percentage of the population with food insecurity in Mecklenburg County more or less than expected based upon the median annual income?
4. Do you think this comparison should be used as a means for celebrating or increasing concern regarding food insecurity in Mecklenburg County? Why or why not?
5. Do you believe that the farmers market program was effective in reducing food insecurity in Mecklenburg County? What other research could you do to find out?

DO THE MATH

PLANNING NOTES

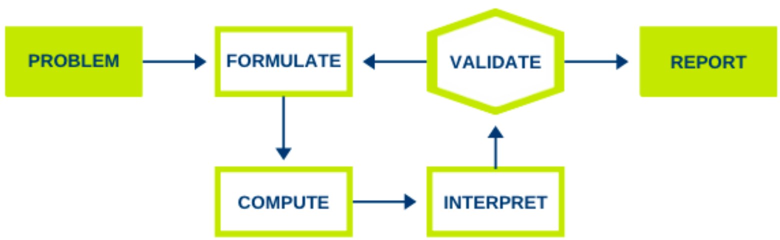
[5] Feeding America Action. *State-By-State Resource: The Impact of Coronavirus on Food Insecurity,* https://feedingamericaaction.org/resources/state-by-state-resource-the-impact-of-coronavirus-on-food-insecurity/

Station F: Micro-Modeling (*20 minutes*)

Instructional Routine: Aspects of Mathematical Modeling

Modeling is the link between the mathematics students learn in school and the problems they will face in college, career, and life. Time spent on modeling in Math 1 is crucial, as it prepares students to use math to handle technical subjects in their further studies, and problem solve and make decisions that adults regularly encounter in their lives.

ASPECTS OF MATHEMATICAL MODELING	***What Is This Routine?*** In activities tagged with this routine, students engage in scaled-back modeling scenarios, for which students only need to engage in a part of a full modeling cycle. For example, they may be selecting quantities of interest in a situation or choosing a model from a list. ***Why This Routine?*** Mathematical modeling is often new territory for both students and teachers. Activities tagged as *Aspects of Mathematical Modeling* offer opportunities to develop discrete skills in the supported environment of a classroom lesson to make success more likely when students engage in more open-ended modeling.



Station F[6]

Funds totaling $191,000 are designated for four schools. The distribution of the funds is to be in proportion to the number of students in each school. Student populations of the four schools are: School A, 386; School B, 1691; School C, 2109; School D, 817.

1. Figure out how much money each school gets.
2. Draw some sort of diagram to scale that helps show visually how the money is divided up among the four schools.
3. Find general formulas for deciding how much money each of the four schools gets in terms of the populations of the schools and the total amount of money to be distributed.

DO THE MATH

PLANNING NOTES

[6] Adapted from Achievethecore.org

Station G: Practice with Rational Bases (*20 minutes*)

In earlier grades, students observed patterns that occur when evaluating expressions containing exponents. The intent of this station is to help students recall these patterns as they prepare for work involving exponents in the next unit.

Station G

Some mathematicians expand expressions involving exponents when they get stuck when evaluating an expression. The expansion of $2^5 \cdot 2^3$ is

$(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2)$. How might expanding help you finish evaluating the expression?[7]

1. Use expansion to evaluate the following expressions involving exponents.
   a. $7^5 \cdot 7^6$
   b. $\left(\frac{2}{3}\right)^3$
   c. $\frac{3^5}{3^2}$

Some mathematicians use these symbolic equations to define exponents rules.

A. $x^n \cdot x^m = x^{n+m}$ B. $(x^n)^m = x^{n \cdot m}$ C. $\frac{x^n}{x^m} = x^{n-m}$ D. $x^{-n} = \frac{1}{x^n}$ E. $x^0 = 1$

2. Match each of the expressions to a rule or rules that could be used to help evaluate it and then evaluate the expression. The first row has been completed for you as an example.

Expression	Letter corresponding to rule or rules above	Evaluate
$2^8 \cdot 2^{-8}$	Rules A and E	$2^8 \cdot 2^{-8} = 2^{(8+-8)} = 2^0 = 1$
$(7^2)^3$		
$\frac{6^5}{6^{-8}}$		
$(3^4)^0$		
11^{-8}		
$(6^{-3})^5$		
$\left(\frac{5}{6}\right)^4\left(\frac{5}{6}\right)^5$		
$\frac{10^5}{10^5}$		

[7] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

DO THE MATH

PLANNING NOTES

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

Reflect on how comfortable your students are asking questions of you and of each other. What can you do to encourage students to ask questions?

Lesson 11: Comparing Graphs

PREPARATION

Lesson Goals	Learning Targets
• Compare key features of graphs of functions and interpret them in context. • Interpret equations of the form $f(x) = g(x)$ in context and recognize that the solutions to such an equation are the x-coordinates of the points where the graphs of f and g intersect. • Interpret statements about two or more functions written in function notation.	• I can compare the features of graphs of functions and explain what they mean in the situations represented. • I can make sense of an equation of the form $f(x) = g(x)$ in terms of a situation and a graph and know how to find the solutions. • I can make sense of statements about two or more functions when they are written in function notation.

Lesson Narrative

In this lesson, students deepen their understanding of functions by comparing representations of several functions relating the same pair of quantities. They analyze two or more graphs simultaneously, interpreting their relative features and their average rates of change in context.

Students also study comparative statements in function notation, such as $A(x) = B(x)$ or $B(10) > A(10)$, and explain them in terms of changes in population, changes in the trends of phone ownership, and the popularity of different television shows.

Students pay close attention to the intersection of two graphs in this lesson. Earlier in their study, they learned that a solution to a system of linear equations in two variables is a point where the graphs of the equations in the system intersect. Here, students recognize that a solution to an equation of the form $f(x) = g(x)$ is an input-output pair that is common to both f and g. They see that a solution to such an equation is the x-coordinate of a point where the graphs of f and g intersect.

Making comparisons involves looking beyond individual pieces of information. To accurately relate the information from multiple representations requires careful and precise use of mathematical language and notation (MP6). Students continue reasoning abstractly and quantitatively (MP2) as they use their analyses of representations of functions to draw conclusions about the quantities in situations.

Share some ways you see this lesson connecting to previous lessons in this unit. What connections will you want to make explicit?

Focus and Coherence

Building On	Addressing
NC.8.F.1: Understand that a function is a rule that assigns to each input exactly one output. • Recognize functions when graphed as the set of ordered pairs consisting of an input and exactly one corresponding output. • Recognize functions given a table of values or a set of ordered pairs.	**NC.M1.A-REI.10:** Understand that the graph of a two variable equation represents the set of all solutions to the equation. **NC.M1.A-REI.11:** Build an understanding of why the x-coordinates of the points where the graphs of two linear, exponential, and/or quadratic equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ and approximate solutions using graphing technology or successive approximations with a table of values. **NC.M1.F-IF.4:** Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; and maximums and minimums. **NC.M1.F-IF.6:** Calculate and interpret the average rate of change over a specified interval for a function presented numerically, graphically, and/or symbolically.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*15 minutes*)
- **Activity 2** (*Optional, 10 minutes*)
- **Activity 3** (*10 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
    - M1.U5.L11 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional*, *5 minutes*)

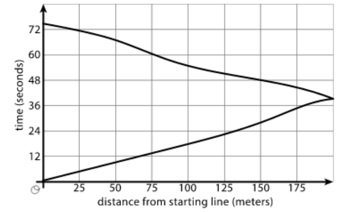
Building On: NC.8.F.1

The purpose of this bridge is for students to review how to interpret coordinates on graphs of a non-function as well as understand that context does not dictate the independent and dependent variables. The relationship between the contexts, the graph, and the individual points is key in setting the students up for success in this lesson.

If there is time and space to discuss, ask students to indicate if they think the graph represents a function or not. If there are students who say yes and no, invite students from each side to say why they think it is or is not a function and then try to persuade the rest of the class to their side. If not all students are persuaded that the graph is not a function, remind students that functions can only have one output for each input, and ask students to look back at their answer to the problem, “Estimate when she was 100 meters from her starting point.” Since that problem has two responses, the graph cannot be a function.

Student Task Statement

Priya is running once around the track. The graph shows her time given how far she is from her starting point.[1]



1. What was her farthest distance from her starting point?
2. Estimate how long it took her to run around the track.
3. Estimate when she was 100 meters from her starting point. How do you know?
4. Estimate how far she was from the starting line after 60 seconds. How do you know?
5. What does the point $(150, 48)$ represent?

DO THE MATH

PLANNING NOTES

Warm-up: Population Growth (*5 minutes*)

Addressing: NC.M1.A-REI.10; NC.M1.F-IF.4	**Building Towards:** NC.M1.A-REI.11

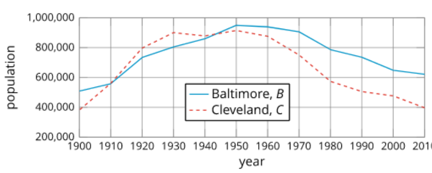
In this warm-up, students compare functions by analyzing graphs and statements in function notation. The work here prepares students to make more sophisticated comparisons later in the lesson.

Step 1

- Give students some quiet work time to read the scenario and respond to the questions**.** Ask students to make their thinking visible by marking up the graph as they answer the questions. Identify students with clear reasoning to call upon during the discussion, intentionally providing opportunities for students who rarely volunteer to share.

Student Task Statement

This graph shows the populations of Baltimore and Cleveland in the 20th century. $B(t)$ is the population of Baltimore in year t. $C(t)$ is the population of Cleveland in year t.



1. Estimate $B(1930)$ and explain what it means in this situation.
2. Here are pairs of statements about the two populations. In each pair, which statement is true? Be prepared to explain how you know.

   a. $B(2000) > C(2000)$ or $B(2000) < C(2000)$

   b. $B(1900) = C(1900)$ or $B(1900) > C(1900)$

3. Were the two cities' populations ever the same? If so, when?

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[1] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

Step 2

- Invite students to share their response to the first question. After students give a reasonable estimate of the population of Baltimore (about 800,000), display the statement $B(1930) = 800,000$ for all to see. Make sure students can interpret it to mean: "In 1930, the population of Baltimore was about 800,000 people."
- Ask students to explain how they knew which statement in each pair of inequalities is true, and how they knew that there were two points in time when Baltimore and Cleveland had the same population.
- Ask students how we could use function notation to express that the populations of Baltimore and Cleveland were equal in 1910. If no students mention $B(1910) = C(1910)$ or $B(t) = C(t)$ for $t = 1910$, bring these up and display these statements for all to see.

DO THE MATH	PLANNING NOTES

Activity 1: Wired or Wireless? (*15 minutes*)

Instructional Routine: Co-Craft Questions (MLR5)
Addressing: NC.M1.A-REI.10; NC.M1.A-REI.11; NC.M1.F-IF.4; NC.M1.F-IF.6

In this activity, students continue to compare two functions by studying graphs and statements in function notation as well as interpreting them in terms of a situation. They also revisit the meaning of a solution to an equation such as $C(t) = 20$, both abstractly and in context, and apply what they know about average rate of change to compare the trend shown by each graph.

Additionally, the activity draws students' attention to the point where the two graphs intersect, its meaning in context, and its corresponding representation in function notation.

Step 1

- Use the *Co-Craft Questions* routine to help students compare and interpret two functions. Display the first two sentences of the student task statement along with the graph, leaving out the questions. If students seem stuck, ask students how many of them have a landline phone at home and how many only have cell phones. If students are unfamiliar with landline phones, explain as needed.
- Ask students to write down possible mathematical questions that could be asked about the situation. Invite students to compare their questions before revealing the activity's questions.
- Listen for and amplify any questions involving the overall shape of the graphs, trends over specific time intervals, and the intersection point of the two graphs. The process of creating mathematical questions, without the pressure of producing answers or solutions, prompts students to make sense of the given information and to activate the language of mathematical questions. This work helps to prepare students to process the actual questions.

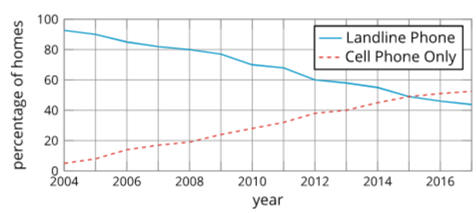
Step 2

- Ask students to arrange themselves in pairs or use visibly random grouping.
- Provide 5 minutes of quiet work time for students to answer the questions in the student task statement and an additional few minutes to share their responses with their partner.
- As student pairs work, circulate and prompt with questions such as:
  - "How would you describe the trends in phone ownership over the years?" (Cell phones are increasing in use, and landlines are decreasing.)
  - "How would you describe the shape of the graph of each function?" (Both are roughly linear. The value of H is decreasing over time, so the graph slants downward. The value of C is increasing over time, so the graph slants upward.)
  - "In what year did about 60% of homes have a landline?" (2012)

Advancing Student Thinking: Students encounter percentages as the output of a function for the first time in this activity. Some students might think that the output of the functions here must be the number of homes, and that they cannot estimate any output values because only percentages are known. Clarify that percent is the unit used in this case, as we are studying how the proportion of the two groups (rather than the actual number of homes in each group) changed over time.

Student Task Statement

$H(t)$ is the percentage of homes in the United States that have a landline phone in year t. $C(t)$ is the percentage of homes with *only* a cell phone. Here are the graphs of H and C.



1. Estimate $H(2006)$ and $C(2006)$. Explain what each value tells us about the phones.
2. What is the approximate value of t when $C(t) = 20$? Explain what that value of t means in this situation.
3. Determine if each equation is true. Be prepared to explain how you know.
   a. $C(2011) = H(2011)$
   b. $C(2015) = H(2015)$
4. Interpret the following statement: "For $t > 2015$, $C(t) > H(t)$." Is the statement true?
5. Between 2004 and 2015, did the percentage of homes with landlines decrease at the same rate at which the percentage of cell-phones-only homes increased? Explain or show your reasoning.

Are You Ready For More?

1. Explain why the statement $C(t) + H(t) \leq 100$ is true in this situation.
2. What value does $C(t) + H(t)$ appear to take between 2004 and 2017? How much does this value vary in that interval?

Step 3

- Facilitate a whole-class discussion focusing on the meaning of equations such as $C(t) = 20$ and $C(2015) = H(2015)$, and on the meaning of the average rate of change of each function.
- Select students to share their responses. Highlight the following points, if not already mentioned in students' explanations:
  - $C(t) = 20$ means "in year t, 20% of homes relied only on cell phones" and, based on the graph of C, the value of t that makes that statement true is 2008.
  - $C(2015) = H(2015)$ means "in year 2015, the percentage of homes with only cell phones and the percentage of homes with landlines are equal." We know this is true because at $t = 2015$ the two graphs intersect, which also means they share the same output value.
  - For $t > 2015$, $C(t) > H(t)$ means "for every year after 2015, the percentage of homes with only cell phones is greater than the percentage of homes with landlines."
  - The average rate change for C is positive because, in the measured interval, the value of C increased overall. The average rate of H is negative because the value of H decreased overall.
  - The average rate of change for C is about 4% per year, and the average rate of change for H is about -4% per year. These rates can be calculated by using estimates for the two vertical intercepts and the intersection point. The fact that the rates of change are similar (with one positive and one negative) can also be estimated visually, without calculating.
- If time permits, discuss with students:
  - "The average rate of change for C is 4% per year, while the average rate of change for H is -3.8% per year, which is very close to -4%. Why might the rate at which one increased be so close to the rate at which the other decreased? Could it be a coincidence?" (One possible explanation is that, as people relied more on cell phones, they relied less on landlines, and they discontinued using landlines around the same time they acquired new cell phones. These people essentially went from one group to the other.)

DO THE MATH	PLANNING NOTES

Activity 2: Audience of TV Shows (*Optional, 10 minutes*)

Instructional Routine: Discussion Supports (MLR8)
Addressing: NC.M1.F-IF.4

Previously, students compared functions by analyzing their graphs on the same coordinate plane. Each graph was a continuous graph.

In this optional activity, students compare functions represented in separate graphs. Each graph is a discrete graph, showing the viewership of three TV shows as functions of the episode number. Students interpret features of the graphs and relate them to descriptions of the shows and to statements in function notation. They use their analyses to draw conclusions about the popularity of the shows and to sketch a possible graph for a fourth TV show.

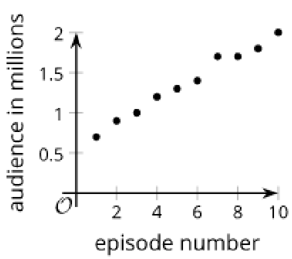
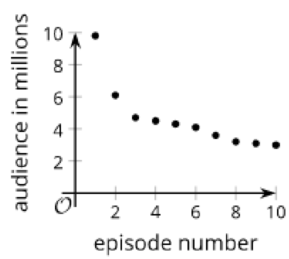
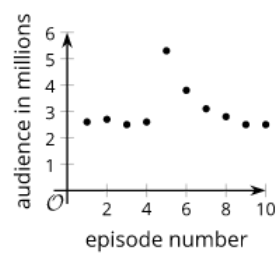
The work here requires students to make sense of quantities and their relationships while attending to their representations (MP2). In sketching a graph that matches a description, students need to be careful about showing correspondence to the quantities in the situation (MP6), including by using appropriate scale and marks.

Step 1

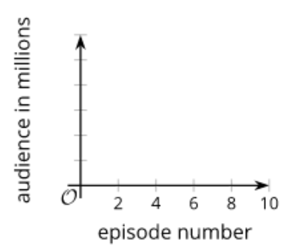
- Have students continue with their partner.
- Give students a few minutes of quiet work time and then time to discuss their thinking with their partner.

Student Task Statement

The number of people who watched a TV episode is a function of that show's episode number. Here are three graphs of three functions—A, B, and C—representing three different TV shows.

Show A	Show B	Show C
		
Description: ________	Description: ________	Description: ________

1. Match each description with a graph that could represent the situation described. One of the descriptions has no corresponding graph.
    a. This show has a good core audience. They had a guest star in the fifth episode that brought in some new viewers, but most of them stopped watching after that.
    b. This show is one of the most popular shows, and its audience keeps increasing.
    c. This show has a small audience, but it's improving, so more people are noticing.
    d. This show started out huge. Even though it feels like it crashed, it still has more viewers than the other two shows.
2. Which is greatest, $A(7)$, $B(7)$, or $C(7)$? Explain what the answer tells us about the shows.
3. Sketch a graph of the viewership of the fourth TV show that did not have a matching graph.



Step 2

- Facilitate a discussion focused on how students made their matches. Ask students to explain how parts of the descriptions and features of the graphs led them to believe that a pair of representations belong together.

Use *Discussion Supports* to support whole-class discussion as students explain how they matched the description to the graph. Display the following sentence frames for all to see: "_____ matches _____ because..." and "I noticed _____, so..." Encourage students to challenge each other when they disagree. This routine will help students explain how they used parts of the descriptions and features of the graphs to find matches.

- Invite students to share their graph of the fourth TV show. Display the graphs for all to see and discuss how the graphs are alike and how they are different. Because each graph is created based on the same description, they should share some common features. If they look drastically different, solicit possible reasons. (Possible explanations include differences in interpretation of the description or in the choice of scale for the vertical axis, errors in reading the description, and plotting errors.)

- If time permits, ask students:

  - "How are the graphs or the functions in this activity different from those in earlier activities?" (They show points rather than lines. The input is episode number, while in other cases, the input was time. Each function is shown on a different coordinate plane.)

  - "How is the work of comparing functions here like comparing functions in earlier activities?" (They all involve comparing the output values of points on a graph, interpreting the points, making sense of verbal descriptions, and some estimating.)

  - "How is the work of comparing functions here different than in earlier activities?" (Here, we are comparing function values across three separate graphs, which is trickier than when the graphs are all on the same coordinate plane, especially because the scales on the vertical axis are different. Figuring out which function has a greater or lesser value was also easier when the functions were represented with lines or curves. It is harder to do with a set of points, especially when they are not in the same image.)

DO THE MATH	PLANNING NOTES

Activity 3: Functions f and g (*10 minutes*)

Instructional Routine: Collect and Display (MLR2)
Addressing: NC.M1.A-REI.11; NC.M1.F-IF.4

In this activity, students compare graphs and statements that represent functions without a context. Because no concrete information is given, students need to rely on their understanding of function notation and points on a graph to make comparisons and to interpret intersections of the graphs.

Previously, students saw equations such as $f(8) = g(8)$ and interpreted them in terms of a situation. Here, they begin to reason more abstractly about statements of the form $f(x) = g(x)$ and relate it to one or more points where the graphs of f and g intersect. They see that a value of x that makes this equation true, or a solution to the equation, is the input value of such an intersection.

Step 1

- Give students quiet work time to respond to each question.

Student Task Statement

1. Here are graphs that represent two functions, f and g.

   Decide which function value is greater for each given input. Be prepared to explain your reasoning.

   a. $f(2)$ or $g(2)$
   b. $f(4)$ or $g(4)$
   c. $f(6)$ or $g(6)$
   d. $f(8)$ or $g(8)$

2. Is there a value of x for which the equation $f(x) = g(x)$ is true? Explain your reasoning.

3. Identify at least two values of x for which the inequality $f(x) < g(x)$ is true.

Step 2

- Facilitate a whole-class discussion by displaying the graphs of f and g for all to see. Invite students to share their responses to the first set of questions. As they point out the greater function value in each pair, mark the point on the graph and write a corresponding statement in function notation: $f(2) < g(2)$, $f(4) = g(4)$, $f(6) > g(6)$, and $f(8) > g(8)$. Emphasize that the function value that is greater in each pair has a higher vertical value on the coordinate plane for the same input value.

In the whole-group discussion, use the *Collect and Display* routine to listen for and collect the language students use to share their interpretations of equations written as $f(x) = g(x)$. Write students' words and phrases on a visual display. Be sure to emphasize words and phrases such as, "the values are equal," "intersect," "horizontal value," "input value," and "common point." Remind students to borrow language from the display as needed. This will help students begin to understand statements of the form $f(x) = g(x)$ and make connections to the graph.

- Discuss if there are values of x that make $f(x) = g(x)$ true and how to tell. Point out that earlier, we wrote $f(4) = g(4)$ because the values of f and g are equal when the input is 4. This means that $(4, f(4))$ and $(4, g(4))$ are coordinates of the same point. So we can interpret an equation such as $f(x) = g(x)$ to mean that the values of f and g are equal when the input is x, and that x must be the horizontal value of the intersection of both graphs, which is a point that they share.

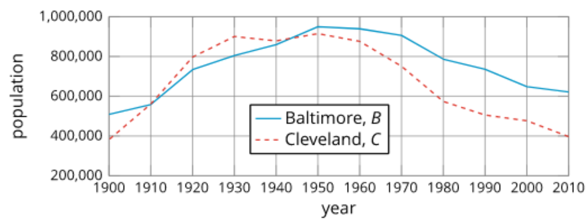
DO THE MATH	**PLANNING NOTES**

Lesson Debrief (*5 minutes*)

The purpose of this lesson is for students to compare key features of graphs and to use function notation to express those ideas precisely. In this debrief, students have another opportunity to make connections between statements made in function notation to verbal comparisons of a graph of two functions. The last two statements concern average rate of change. Discussing "how can we tell" for these statements provides a good reminder that we can use slope (even on nonlinear graphs) to compare rates of change. Expressing this in function notation, however, is more difficult. Teachers may want to skip the function notation column for these statements, or leave the function notation as a challenge for the class or for individual students.

Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion.

Refer back to the population graphs for Baltimore and Cleveland from 1900 to 2010, which students saw in the warm-up. Display the graphs for all to see.



Present students with the following statements, one at a time, about the populations of the two cities. Tell students that their job is to explain how they could tell from the graphs that each statement is true, and to translate each verbal description into a statement with the same meaning but written in function notation.

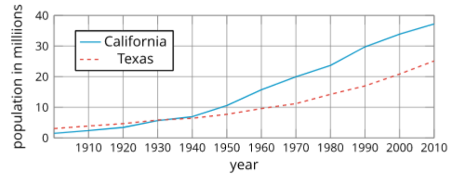
Consider using a three-column graphic organizer (as shown here and in the Lesson Summary) to organize students' responses.

What can we tell about the populations?	How can we tell?	How can we convey this with function notation?
In 2010, Baltimore had more people than Cleveland.	For an input value of 2010, the point on graph B has a higher vertical value than the point on graph C.	$B(2010) > C(2010)$
Baltimore and Cleveland had the same population twice in the past century, in 1910 and around 1944.	The two graphs cross at the horizontal values 1910 and approximately 1944.	$B(1910) = C(1910)$ and $B(1944) = C(1944)$
After the mid-1940s, Cleveland has had a smaller population than Baltimore.	For input values greater than 1944, graph C stays below graph B.	For $x > 1944$, $C(x) < B(x)$
In the first half of the 20th century, the population of Cleveland grew at a faster rate than that of Baltimore.	Draw a straight line connecting the points $(1900, B(1900))$ and $(1950, B(1950))$. Do the same thing for graph C. The slope of the line for graph C is steeper.	$\frac{C(1950)-C(1900)}{1950-1900} > \frac{B(1950)-B(1900)}{1950-1900}$
Since 1950, the population of Cleveland has dropped at a faster rate than that of Baltimore.	Draw a straight line connecting the points $(1950, B(1950))$ and $(2010, B(2010))$. Do the same thing for graph C. The slope of the line for graph C is steeper (more negative).	$\frac{C(1020)-C(1950)}{2010-1950} < \frac{B(2010)-B(1950)}{2010-1950}$ Since both quantities are negative, this means that C is dropping at a faster rate.

PLANNING NOTES

Student Lesson Summary and Glossary

Graphs are very useful for comparing two or more functions. Here are graphs of functions C and T, which give the populations (in millions) of California and Texas in year x.



What can we tell about the populations?	How can we tell?	How can we convey this with function notation?
In the early 1900s, California had a smaller population than Texas.	The graph of C is below the graph of T when x is 1900.	$C(1900) < T(1900)$
Around 1935, the two states had the same population of about 5 million people.	The graphs intersect at about $(1935, 5)$.	$C(1935) = 5$ and $T(1935) = 5$, and $C(1935) = T(1935)$
After 1935, California has had more people than Texas.	When x is greater than 1935, the graph of $C(x)$ is above that of $T(x)$.	$C(x) > T(x)$ for $x > 1935$
Both populations have increased over time, with no periods of decline.	Both graphs slant upward from left to right.	
From 1900 to 2010, the population of California has risen faster than that of Texas. California had a greater average rate of change.	If we draw a line to connect the points for 1900 and 2010 on each graph, the line for C has a greater slope than that for T.	

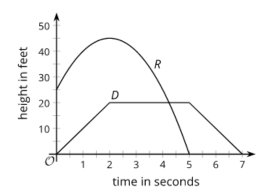
Cool-down: A Toy Rocket and a Drone Again (*5 minutes*)

Addressing: NC.M1.A-REI.11; NC.M1.F-IF.4

Cool-down Guidance: Points to Emphasize
Share a few examples of student work in a subsequent lesson to highlight misconceptions.

Cool-down

Functions R and D give the height, in feet, of a toy rocket and a drone, t seconds after they are released. Here are the graphs of R (for the rocket) and D (for the drone).



1. Which of the inequalities is true: $R(2) > D(2)$ or $R(2) < D(2)$?
2. What was the height of the drone when the toy rocket hit the ground?
3. For what value of t is $R(t) = D(t)$ true? What does this tell you about the drone and the toy rocket?
4. Find $R(2) - D(2)$ and describe what it means given the context of the situation.

Student Reflection:

Mathematical vocabulary helps us to understand the work better in order to engage with it. What is a new mathematical term you learned today, and how did it contribute to your ability to complete your work?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

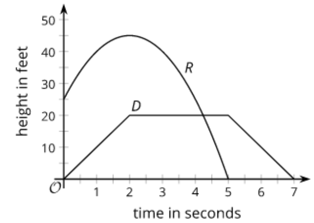
TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

What did you say, do, or ask during the lesson debrief that helped students be clear on the learning of the day? How did understanding the cool-down of the lesson before you started teaching today help you synthesize that learning?

Practice Problems

1. Functions R and D give the height, in feet, of a toy rocket and a drone, t seconds after they are released.

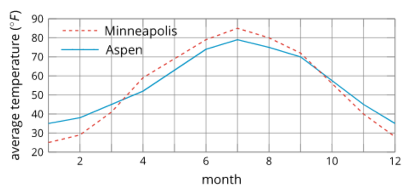
   Here are the graphs of R (for the rocket) and D (for the drone).

   Write each statement in function notation:

   a. At 3 seconds, the toy rocket is higher than the drone.

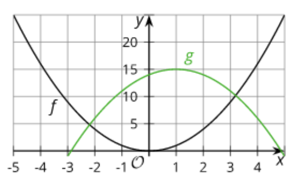
   b. At the start, the toy rocket is 25 feet above the drone.

2. $A(t)$ is the average high temperature in Aspen, Colorado, t months after the start of the year. $M(t)$ is the average high temperature in Minneapolis, Minnesota, t months after the start of the year. Temperature is measured in degrees Fahrenheit.

   Which function had the higher average rate of change between the beginning of January ($t = 1$)and middle of March $(t \approx 3.5)$? What does this mean about the temperature in the two cities?

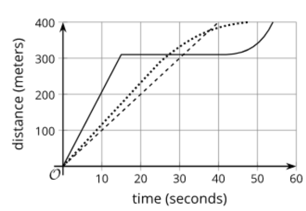
3. Here are two graphs representing functions f and g.

   Select **all** statements that are true about functions f and g.

   a. $f(0) > g(0)$
   b. There are two values of x where $f(x) = g(x)$.
   c. If $g(x) = 15$, then $x = 1$.
   d. $f(-3) > g(4)$

4. The three graphs represent the progress of three runners in a 400-meter race.

   The solid line represents runner A. The dotted line represents runner B. The dashed line represents runner C.

   a. One runner ran at a constant rate throughout the race. Which one?

   b. A second runner stopped running for a while. Which one? During which interval of time did that happen?

   c. Describe the third runner's race. Be as specific as possible.

   d. Who won the race? Explain how you know.

5. Function f is represented by $f(x) = 5(x + 11)$.

    a. Find $f(-2)$.

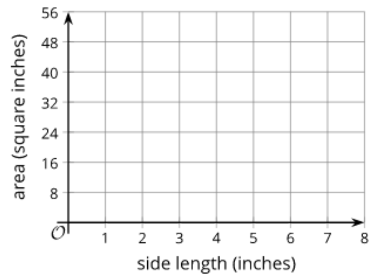
    b. Find the value of x such that $f(x) = 90$ is true.

    (From Unit 5, Lesson 5)

6. Function A gives the area, in square inches, of a square with side length x inches.

    a. Complete the table.

x	0	1	2	3	4	5	6
$A(x)$							



    b. Represent function A using an equation.

    c. Sketch a graph of function A.

    (From Unit 5, Lesson 4)

7. Solve the following system of equations. In which quadrant is the point that represents the solution on the coordinate plane? Explain or show your reasoning.

$$\begin{cases} y = 3x + 6 \\ x = 2y - 7 \end{cases}$$

    (From Unit 3)

8. On a math test, there are multiple choice questions worth 4 points and open-ended questions worth 6 points. Mr. Pills' first period class won a competition, and each student had 10 extra credit points added to their test score!

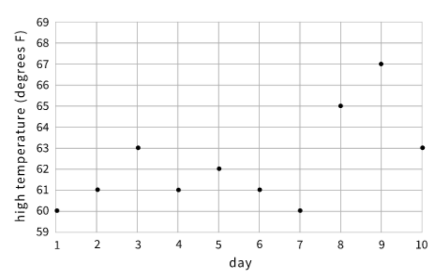
    a. If Mai answered 13 multiple choice questions correctly and earned a total score of 92, how many open-ended questions did she answer correctly?

    b. If Priya answered 4 open-ended questions correctly and earned a total score of 94, how many multiple choice questions did she answer correctly?

    c. Write an expression that will represent a students' score in first period who answers *m* multiple choice questions and *e* open-ended questions correctly.

    (From Unit 2)

9. The graph below shows the high temperatures in a city over a 10-day period.[2]



    a. What was the highest temperature in the city during the 10-day period?

    b. What was the temperature on Day 3?

    c. On what day was the temperature 65°?

    d. What does the point (5, 62) represent on the graph?

    (Addressing NC.8.F.1)

[2] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

Lesson 12: Domain and Range (Part One)

PREPARATION

Lesson Goals	Learning Targets
• Given a description of a function that represents a situation, determine a reasonable domain and range. • Understand that the domain of a function is the set of all possible inputs and the range is the set of all possible outputs.	• When given a description of a function in a situation, I can determine a reasonable domain and range for the function. • I know what is meant by the "domain" and "range" of a function.

Lesson Narrative

Before this lesson, students have considered (even if only peripherally) input and output values that would make sense in the context of a function. Some examples:

- When analyzing the cost of buying bagels, it was intuitive to consider only positive whole numbers for the input.
- When studying the total number of barks of a dog as a function of time, it was natural to consider only positive whole numbers for the output.
- When looking at the height of a projectile as a function of time, it made sense to limit the input from the launch of the object to the moment the object hits the ground.

In this lesson and the next one, students focus their attention on possible input and output values, framing them as the **domain** and **range** of a function. In this lesson, they identify the domain and range of functions and describe them using words, lists of numbers, or inequalities (if appropriate). In the next lesson, students will relate the domain and range of a function to features of its graph.

Students' analyses of inputs and outputs continue to be grounded in context, allowing many chances to reason quantitatively and abstractly (MP2). The insights students gain here will help them later in the unit and throughout the course, as they make sense of other kinds of functions—exponential and quadratic. These understandings will also expand students' capacity to model with mathematics.

What teaching strategies will you be focusing on during this lesson?

Focus and Coherence

Building On	Addressing
NC.8.F.1: Understand that a function is a rule that assigns to each input exactly one output. ● Recognize functions when graphed as the set of ordered pairs consisting of an input and exactly one corresponding output. ● Recognize functions given a table of values or a set of ordered pairs.	**NC.M1.A-REI.10:** Understand that the graph of a two variable equation represents the set of all solutions to the equation. **NC.M1.F-IF.5:** Interpret a function in terms of the context by relating its domain and range to its graph and, where applicable, to the quantitative relationship it describes.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*15 minutes*)
- **Activity 2** (*10 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U5.L12 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional*, *5 minutes*)

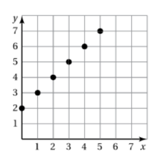
Instructional Routine: Notice and Wonder

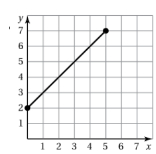
Building On: NC.8.F.1

The purpose of this bridge is to have students *Notice and Wonder* about the differences in the input of the two functions and how this impacts the possible outputs. Students may speculate about the quantities the first graph may represent, such as number of people or number of items. Looking at the structure (MP7) of the possible outputs for given inputs sets up the concepts of discrete and continuous domains in this lesson.

Student Task Statement

Observe the two graphs. What do you notice? What do you wonder?





DO THE MATH

PLANNING NOTES

Warm-up: Number of Barks (*5 minutes*)

Addressing: NC.M1.F-IF.5

This warm-up prompts students to consider possible input and output values for a familiar function in a familiar context. The work here prepares students to do the same in other mathematical contexts and to think about domain and range in the rest of the lesson.

Step 1

- Ask students to arrange themselves in pairs or use visibly random grouping.
- Provide 2–3 minutes for students to work through the two questions.

Student Task Statement

Earlier, you saw a situation where the total number of times a dog has barked was a function of the time, **in seconds**, after its owner tied its leash to a post and left to go into a store. Less than 3 minutes after he left, the owner returned, untied the leash, and walked away with the dog.

1. Could each value be an input of the function? Be prepared to explain your reasoning.

   15 $84\frac{1}{2}$ 300

2. Could each value be an output of the function? Be prepared to explain your reasoning.

   15 $84\frac{1}{2}$ 300

Step 2

- Invite students to share their responses and reasoning. Highlight explanations that make a convincing case as to why values beyond 180 could not be inputs for this function and why fractional values could not be outputs.
- Some students may argue that 300 could be an input because "300 seconds after the dog's owner walked away" is an identifiable moment, even though the dog and its owner have walked away and may no longer be near the post. Acknowledge that this is a valid point, and that it highlights the need for a function to be more specifically defined in terms of when it "begins" and "ends." If time permits, solicit some ideas on how this could be done.
- Tell students that, in this lesson, they will think more about values that make sense as inputs and outputs of functions.

DO THE MATH	PLANNING NOTES

Activity 1: Possible or Impossible? (*15 minutes*)

Instructional Routine: Discussion Supports (MLR8) - Responsive Strategy
Addressing: NC.M1.F-IF.5

Students continue to think about reasonable input values for functions based on the situation that they represent. They are given three functions and a list of rational values. For each function, they determine which values make sense as inputs and why. The idea of the domain of a function is then introduced.

Step 1

- Ask students to arrange themselves in pairs or use visibly random grouping.
- For each function defined in their activity statement, ask students to sort the numbers into two groups, "possible inputs" or "impossible inputs," based on whether or not the function could take the number as an input. Students will circle the numbers that are “possible inputs” and cross out the numbers that are “impossible inputs.”
- Some students may be unfamiliar with camps and may not know that other units besides Fahrenheit and Celsius are used to measure temperature. Students may not know that $0°K$ or $-273.15°C$ is absolute zero temperature, the lowest possible temperature. Consider sharing this information with them in preparation for them describing the domain of function k. Provide a brief orientation of these terms, if needed, along with visuals.

RESPONSIVE STRATEGY

After sorting possible inputs for the first function, provide the class with the following sentence frames to help groups respond to each other: “____ is a possible/impossible input because . . .” and “I agree/disagree because . . . .” When monitoring discussions, revoice student ideas to demonstrate mathematical language. This will help students listen and respond to each other as they explain how they sorted the numbers.

Discussion Supports (MLR8)

- Consider asking groups to pause after sorting possible inputs for the first function and to discuss their decisions with another group. If the two groups disagree on where a number belongs, they should discuss until they reach an agreement, and then continue with the rest of the activity.

Monitoring Tip: As students sort the numbers and discuss their thinking in groups, listen for their reasons for classifying a number one way or another. Identify students who can correctly and clearly articulate why certain numbers are or are not possible inputs and let them know that they may be asked to share later. Include at least one student who does not typically volunteer.

Student Task Statement

Decide whether each of the following numbers is a possible input for the functions described here. Sort the numbers into two groups—possible inputs and impossible inputs. Record your sorting decisions in the table.

Functions	Circle the <u>possible inputs</u> and cross out the <u>impossible inputs</u>
1. The area of a square, in square centimeters, is a function of its side length, s, in centimeters. The equation $A(s) = s^2$ defines this function.	-3 9 $\frac{3}{5}$ 15 0.8 4 0 $\frac{25}{4}$ 0.001 -18 6.8 72

2.	A tennis camp charges $40 per student for a full-day camp. The camp runs only if at least 5 students sign up, and it limits the enrollment to 16 campers a day. The amount of revenue, in dollars, that the tennis camp collects is a function of the number of students that enroll. The equation $R(n) = 40n$ defines this function.	-3 0.8 0.001	9 4 -18	$\frac{3}{5}$ 0 6.8	15 $\frac{25}{4}$ 72
3.	The relationship between temperature in Celsius and the temperature in Kelvin can be represented by a function k. The equation $k(c) = c + 273.15$ defines this function, where c is the temperature in Celsius and $k(c)$ is the temperature in Kelvin.	-3 0.8 0.001	9 4 -18	$\frac{3}{5}$ 0 6.8	15 $\frac{25}{4}$ 72

Step 2

- Invite previously chosen students to share their results. Record and display for all to see the values students considered possible and impossible inputs for each function. Discuss any remaining disagreements students might have about particular values.
- Tell students that we call the set of *all* possible input values of a function the domain of the function.
- Ask students, "How would you describe the domain for each function?" Record and display the description that students give for each function, making sure that the descriptions are complete. For example:
  - Area: s, the input of function A can be any value equal to or greater than 0 ($s \geq 0$). The side length can be 0 or any positive number, including irrational numbers. There may be a debate over whether 0 is a possible length of a square. Either side of the debate should be accepted as long as the connection between the input and the side length of a square is made correctly.
  - Tennis camp: n, the input of function R can be any whole-number value that is at least 5 and at most 16 ($5 \leq n \leq 16$). The number of campers cannot be fractional.
  - Temperature: c, the input of function k can be any value that is greater than -273.15 ($-273.15 < c < \infty$).

DO THE MATH

PLANNING NOTES

Activity 2: What About the Outputs? (*10 minutes*)

Instructional Routine: Critique, Correct, Clarify (MLR3) - Responsive Strategy	
Addressing: NC.M1.A-REI.10; NC.M1.F-IF.5	

Earlier, students learned that the domain of a function refers to the set of all possible inputs. In this activity, students are introduced to the range of a function and examine it in terms of a situation. They begin to consider how the domain and range of a function are related to the features of its graph and explore the difference between a discrete and a continuous function.

RESPONSIVE STRATEGY

Leverage choice around perceived challenge. Invite students to analyze either the area function or the revenue function.

Supports accessibility for: Organization; Attention; Social-emotional skills

Step 1

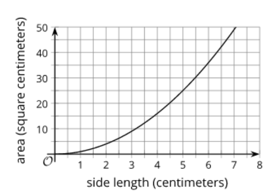
- Keep students in pairs.
- Give students 2 minutes of quiet work time and then 2 minutes to share their responses with their partner.

Advancing Student Thinking: Some students may mistakenly associate the domain and range of a function with the horizontal and vertical values that are visible in a graphing window, or with the upper and lower limits of the scale of each axis on a coordinate plane. For example, they may think that the range of the area function, A, includes only values from 0 to 50 because the scale on the vertical axis goes from 0 to 50. Ask these students if it is possible to use a different scale on each axis or, if the function is graphed using technology, to adjust the graphing window. Clarify that the domain and range should be considered in terms of a situation rather than the graphing boundaries.

Student Task Statement

In an earlier activity, you saw a function representing the area of a square (function A) and another representing the revenue of a tennis camp (function R). Refer to the descriptions of those functions to answer these questions.

1. Here is a graph that represents function A, defined by $A(s) = s^2$, where s is the side length of the square in centimeters.



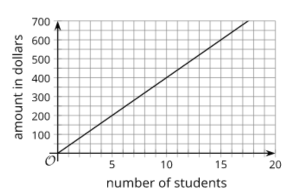
   a. Name three possible input-output pairs of this function.

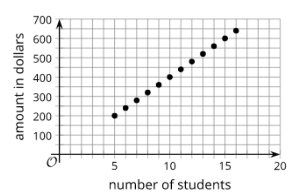
   b. Earlier we described the set of all possible input values of A as "any number greater than or equal to 0." How would you describe the set of all possible output values of A?

2. Function R is defined by $R(n) = 40n$, where n is the number of campers.

   a. Is 20 a possible output value in this situation? What about 100? Explain your reasoning.

   b. Here are two graphs that relate the number of students and camp revenue in dollars. Which graph could represent function R? Explain why the other one could not represent the function.





   c. Describe the set of all possible output values of R.

Are You Ready For More?

If the camp wishes to collect at least $500 from the participants, how many students can they have? Explain how this information is shown on the graph.

Step 2

- Invite students to share their descriptions of the possible outputs for question 1. Explain that we call the set of *all* possible output values of a function the range of the function. Emphasize that the range of a function depends on its domain (or all possible input values). For example, for the area of the square, the range—all the possible values of $A(s)$—includes all numbers that are at least 0.
- Next, focus the discussion on function R.
  - Ask students to explain which values could or could not be the outputs of R and which of the two graphs represent the function.
  - For the revenue of the tennis camp, the range—all the possible values of $R(n)$—includes positive multiples of 40 that are at least 200 and at most 640.
  - Clarify that although the graph showing only points more accurately reflects the domain and range of the function, plotting those points could be pretty tedious. We could use a line graph to represent the function, as long as we specify or are clear that only whole numbers from 5–16 are in the domain, and only multiples of 40 are in the range.
  - Share with students that a function like R is called a discrete function, meaning that its inputs and outputs are specific, distinct values.

RESPONSIVE STRATEGY

Before students share their descriptions of the possible output values of A, present an incorrect response and explanation. For example, "The outputs of A are numbers from 0 to 50 because I looked on the vertical axis and saw that the graph reaches up to 50." Ask students to identify the error, critique the reasoning, and write a correct explanation. As students discuss with a partner, monitor for students who clarify that the output values are not restricted by the graphing boundaries shown. This helps students evaluate, and improve upon, the written mathematical arguments of others, as they discuss the range of a function.

Critique, Correct, Clarify (MLR3)

DO THE MATH

PLANNING NOTES

Lesson Debrief (*5 minutes*)

The purpose of this lesson is to introduce students to the concepts of domain and range in real-world contexts, considering what values are reasonable inputs for a function, and what values are reasonable outputs.

Choose whether students should first have an opportunity to reflect in their workbooks or talk through the questions in the organizer with a partner.

Tell students that function q gives the number of minutes a person sleeps as a function of the number of hours they sleep in a 24-hour period. Display the blank graphic organizer that is in the Student Workbook.

	In the domain?	In the range?
Negative values	No	No
0	Yes	Yes
Values less than 1	Yes	Yes
24	Yes	Yes
25	No	Yes
60	No	Yes
Fractions	Yes	Yes
Values greater than 480	No	Yes
1,500	No	No

PLANNING NOTES

Ask students to decide whether each value or set of values described in the first column could be in the domain and in the range of the function. They should be prepared to explain their decisions (some of which may depend on the assumptions they made about the situation).

Once the class completes the organizer (an example is shown above), give students a moment to come up with a holistic description of the domain and range of this function.

Student Lesson Summary and Glossary

The **domain** of a function is the set of all possible input values. Depending on the situation represented, a function may take all numbers as its input or only a limited set of numbers.

Domain: The domain of a function is the set of all of its possible input values.

- Function A gives the area of a square, in square centimeters, as a function of its side length, s, in centimeters.
  - The input of A can be 0 or any positive number, such as 4, 7.5, or $\frac{19}{3}$. It cannot include negative numbers because lengths cannot be negative. This means the domain of A includes 0 and all positive numbers ($s \geq 0$).
- Function q gives the number of buses needed for a school field trip as a function of the number of people, n, going on the trip.
  - The input of q can be 0 or positive whole numbers because a negative or fractional number of people doesn't make sense. If the number of people at a school is 120, then the domain is limited to all non-negative whole numbers up to 120 ($0 \leq n \leq 120$).
- Function v gives the total number of visitors to a theme park as a function of days, d, since a new attraction was open to the public.
  - The input of v can be positive or negative. A positive input means days since the attraction was open, and a negative input means days before the attraction was open.
  - The input can also be whole numbers or fractional. The statement $v(17.5)$ means 17.5 days after the attraction was open.
  - This means that the domain of v includes all numbers. If the theme park had been opened for exactly one year before the new attraction was open, then the domain would be all numbers greater than -365 (or $d \geq -365$).

The **range** of a function is the set of all possible output values. Once we know the domain of a function, we can determine the range that makes sense in the situation.

Range: The range of a function is the set of all of its possible output values.

- The output of function A is the area of a square in square centimeters, which cannot be negative but can be 0 or greater, not limited to whole numbers. The range of A is 0 and all positive numbers.
- The output of q is the number of buses, which can only be 0 or positive whole numbers. If there are 120 people at the school, however, and if each bus could seat 30 people, then only up to 4 buses are needed. The range that makes sense in this situation would be any whole number that is at least 0 and at most 4.
- The output of function v is the number of visitors, which cannot be fractional or negative. The range of v therefore includes 0 and all positive whole numbers.

Cool-down: Community Service (*5 minutes*)

Addressing: NC.M1.F-IF.5

Cool-down Guidance: More Chances
Students will have more opportunities to understand the mathematical ideas in this cool-down, so there is no need to slow down or add additional work to the next lessons. Instead, use the results of this cool-down to provide guidance for what to look for and emphasize over the next several lessons to support students in advancing their current understanding.

Cool-down

Diego's club earns money for charity when members of the club perform community service after school. For each student who does community service, the club earns $5. There are 12 students in the club.

The total dollar amount earned, E, is a function of the number of members who perform community service, n.

1. Is 5 a possible input value? Why or why not?
2. Is 24 a possible output value? Why or why not?
3. Describe the domain of this function.
4. Describe the range of this function.

If you get stuck, consider creating a table or a graph.

Student Reflection:

What mathematical questions might you ask in order to find a reasonable domain and range?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

What was the best question you asked students today? Why would you consider it the best one based on what students said or did?

Practice Problems

1. The cost for an upcoming field trip is \$30 per student. The cost of the field trip C, in dollars, is a function of the number of students x.

   Select **all** the possible outputs for the function defined by $C(x) = 30x$.

   a. 20
   b. 30
   c. 50
   d. 90
   e. 100

2. A rectangle has an area of 24 cm^2. Function f gives the length of the rectangle, in centimeters, when the width is w cm.

   Determine if each value, in centimeters, is a possible input of the function.

   3 0.5 48 -6 0

3. Select **all** the possible input-output pairs for the function $y = -3x^2 - 4$.

   a. $(-1, -1)$
   b. $(-2, 8)$
   c. $(1, -7)$
   d. $(0, -4)$
   e. $(3, -23)$
   f. $(4, -52)$

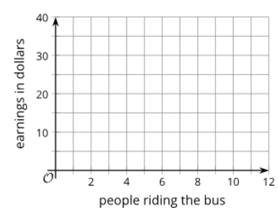
   Is 0 in the range of this function? How do you know?

   Describe the range of this function.

4. A small bus charges \$3.50 per person for a ride from the train station to a concert. The bus will run if at least three people take it, and it cannot fit more than 10 people.

   Function B gives the amount of money that the bus operator earns when n people ride the bus.

   a. Identify all numbers that make sense as inputs and outputs for this function.
   b. Sketch a graph of B.



5. To raise funds for a trip, members of a high school math club are holding a game night in the gym. They sell tickets at \$5 per person. The gym holds a maximum of 250 people. The amount of money raised is a function of the number of tickets sold.

   Which statement accurately describes the domain of the function?

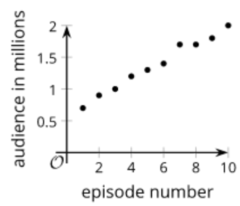
   a. all numbers less than 250
   b. all integers
   c. all positive integers
   d. all positive integers less than or equal to 250

6. The graphs show the audience, in millions, of two TV shows as a function of the episode number.

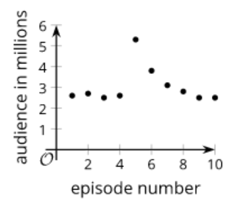
   For each show, try to pick two episode numbers between which the function has a negative average rate of change. Either estimate this average rate of change, or explain why it is not possible.

   (From Unit 5, Lesson 11)

Show A



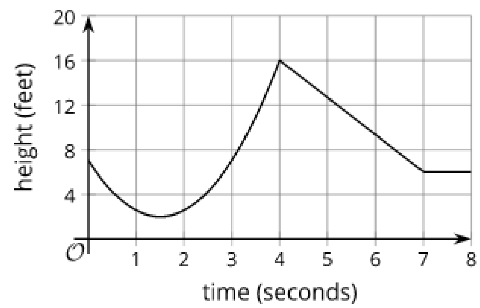
Show C



7. Match each feature of the graph with the corresponding coordinate point.

If the feature does not exist, choose "none."

a. maximum b. minimum c. vertical intercept d. horizontal intercept	i. $(0, 7)$ ii. $(1.5, 2)$ iii. $(4, 16)$ iv. none



(From Unit 5, Lesson 6)

8. Two functions are defined by the equations $f(x) = 5 - 0.2x$ and $g(x) = 0.2(x + 5)$.

Select **all** statements that are true about the functions.

a. $f(3) > 0$
b. $f(3) > 5$
c. $g(-1) = 0.8$
d. $g(-1) > 0.5$
e. $f(0) = g(0)$

(From Unit 5, Lesson 5)

9. The graph of function f passes through the coordinate points $(0, 3)$ and $(4, 6)$.

Use function notation to write the information each point gives us about function f.

(From Unit 5, Lesson 3)

10. Here is a system of equations:

$$\begin{cases} 7x - 4y = -11 \\ 7x + 4y = -59 \end{cases}$$

Would you rather use subtraction or addition to solve the system? Explain your reasoning.

(From Unit 3)

11. The videography team entered a contest and won a monetary prize of $1,350. Which expression represents how much each person would get if there were x people on the team?

a. $\frac{1350}{x}$

b. $1350 + x$

c. $\frac{1350}{5}$

d. $1350 - x$

(From Unit 2)

Lesson 13: Domain and Range (Part Two)

PREPARATION

Lesson Goals	Learning Target
• Given a description of a function that represents a situation, determine a reasonable domain and range. • Practice interpreting key features of graphs in terms of the quantities represented.	• When given a description of a function in a situation, I can determine a reasonable domain and range for the function.

Lesson Narrative

Previously, students reasoned about the domain and range of a function based on descriptions of the situation it represents. In this lesson, students use graphs to learn about the domain and range of functions, while continuing to think about what values are reasonable in the given situations.

Students learn to look for graphical features that would help them identify restrictions to the input or output. For example, they recognize that maximums and minimums, intercepts, and gaps on the graph can be quite informative. They also see that discrete points or breaks in a graph suggest that not all values can be in the domain or range, and that a graph may only partially represent a function. The work here offers opportunities to look for and make use of structure (MP7).

As they examine graphs against situations and vice versa, students practice making sense of quantities (MP1) and reasoning concretely and abstractly (MP2). When describing domain and range, students also practice attending to precision by minding relevant details in the graphs and descriptions of functions (MP6).

Share some ways you see this lesson connecting to previous lessons in this unit. What connections will you want to make explicit?

Focus and Coherence

Building On	Addressing
NC.M1.F-IF.2: Use function notation to evaluate linear, quadratic, and exponential functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	**NC.M1.A-REI.10:** Understand that the graph of a two variable equation represents the set of all solutions to the equation. **NC.M1.F-IF.4:** Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; and maximums and minimums. **NC.M1.F-IF.5:** Interpret a function in terms of the context by relating its domain and range to its graph and, where applicable, to the quantitative relationship it describes.

Agenda, Materials, and Preparation

- **Warm-up** (*5 minutes*)
- **Activity 1** (*20 minutes*)
- **Activity 2** (*10 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U5.L13 Cool-down (print 1 copy per student)

LESSON

Warm-up: Unlabeled Graphs (*5 minutes*)

Instructional Routines: Which One Doesn't Belong?; Round Robin	
Building On: NC.M1.F-IF.4	

This warm-up prompts students to carefully analyze and compare the properties of four graphs using the *Which One Doesn't Belong?* routine. Each graph represents a function, but no labels or scales are shown on the coordinate axes, so students need to look for and make use of the structure of the graphs in determining how each one is like or unlike the others (MP7).

In making comparisons, students have a reason to use language precisely (MP6), especially mathematical terms that describe features of graphs or properties of functions.

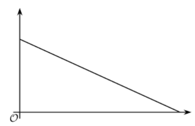
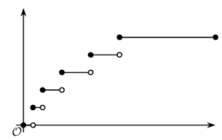
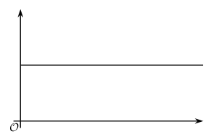
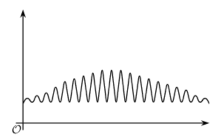
Step 1

- Ask students to arrange themselves in small groups or use visibly random grouping.
- Display the graphs for all to see.
- Give students 1 minute of quiet think time and then time to share their thinking with their small group using the *Round Robin* routine. Ask each student to share with their small group their reasoning as to why a particular graph does not belong, and then ask the group to work together to find at least one reason each item doesn't belong.

ROUND ROBIN	***What Is This Routine?*** In small groups, students take turns sharing their rough draft response to an open-ended question while other group members refrain from comments or questions. A prop can be passed within each group to indicate whose turn it is to talk. After each student has had a turn to share, the group can ask questions of each other; then the teacher selects students to share with the whole class what their group members said. ***Why This Routine?*** Engaging a group of students in collaborative problem solving, with equitable inclusion of ideas, can be challenging due to normative social status issues that place higher value on some students' contributions over others. *Round Robin* allows all students to include their rough draft ideas for solving an open-ended problem without a subset of students dominating the conversation. Knowing all ideas will be shared should motivate all students to try at least one strategy to solve a problem on their own, critical for making sense of problems and persevering in solving them (MP1). The active sharing and listening involved in this routine also provides opportunity for constructing and critiquing viable arguments (MP3).

Student Task Statement

Which one doesn't belong? Explain your reasoning.

a. 	b. 
c. 	d. 

Step 2

- Ask each group to share one reason why a particular item does not belong. Record and display the responses for all to see. Encourage students to use relevant mathematical vocabulary in their explanations, and ask students to explain the meaning of any terminology that they do use, such as "intercepts," "minimum," or "linear functions."
- After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which one does not belong, attend to students' explanations and ensure the reasons given are correct.
- By now students are well aware that certain features of a graph have special significance in that they tell us something about the quantities or relationships in the situation. Tell students that features of graphs can also help us understand the domain and range of a function. We will explore this in the lesson.

DO THE MATH	PLANNING NOTES

Activity 1: Time on the Swing (*20 minutes*)

Instructional Routine: Discussion Supports (MLR8) - Responsive Strategy
Addressing: NC.M1.F-IF.4; NC.M1.F-IF.5

In this activity, students are given the same four graphs they saw in the warm-up and four descriptions of functions, and then are asked to match them. All of the functions share the same context. Students then use these features to reason about the likely domain and range of each function.

To make the matches, students analyze and interpret features of the graphs, looking for and making use of structure in the situation and in the graphs (MP7). Students reason quantitatively and abstractly as they connect verbal and graphical representations of functions and as they think about the domain and range of each function (MP2).

Step 1

- Ask students to imagine a child getting on a swing, swinging for 30 seconds, and then getting off the swing. Explain that they will look at four functions that can be found in this situation. Their job is to match verbal descriptions and graphs that define the same functions, and then to think about reasonable domain and range for each function. Tell students they will need additional information for the last question.

Step 2

- Arrange students in pairs or use visibly random grouping.
- Give students a few minutes of quiet time to think about the first two questions, and then time to discuss their thinking with their partner.

Monitoring Tip: Monitor for the different possible ways students may reason about each match:
- The swing goes up and down while the child is swinging, so D could be a graph for function h.
- The time left on the swing decreases as the time on the swing increases, so A is a possible graph for function r.
- The distance of the swing from the top beam doesn't change, so C is a possible graph for function d.
- The total number of times the swing is pushed must be a counting number and cannot be fractional. Graph B has multiple pieces and each one could represent the total number of pushes for certain intervals of time, so B is a possible graph of s.

Advancing Student Thinking: Some students may struggle to match the descriptions and the graphs because they confuse the independent and dependent variables and think that, in each situation, time is represented by the vertical axis. Encourage them to re-read the activity statement, clarify the input and output in each situation, label the horizontal axes with the input, and then try interpreting the graphs again.

Student Task Statement

A child gets on a swing in a playground, swings for 30 seconds, and then gets off the swing.

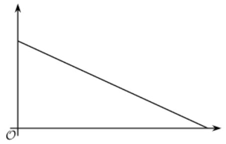
1. Here are descriptions of four functions relating to the situation and four graphs representing them.

   The independent variable in each function is time, measured in seconds.

   Match each function with a graph that could represent it. Then, label the axes with the appropriate variables. Be prepared to explain how you made your matches.

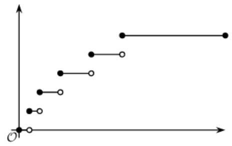
- Function h: The height of the swing, in feet, as a function of time since the child gets on the swing
- Function r: The amount of time left on the swing as a function of time since the child gets on the swing
- Function d: The distance, in feet, of the swing from the top beam (from which the swing is suspended) as a function of time since the child gets on the swing
- Function s: The total number of times an adult pushes the swing as a function of time since the child gets on the swing

Graph A
Function: ____



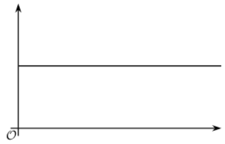
Domain: Range:

Graph B
Function: ____



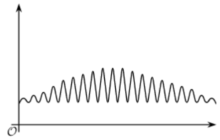
Domain: Range:

Graph C
Function: ____



Domain: Range:

Graph D
Function: ____



Domain: Range:

2. On each graph, mark one or two points that—if you had the coordinates—could help you determine the domain and range of the function. Be prepared to explain why you chose those points.

Step 3

- Invite students to share their matching decisions and explanations as to how they know each pair of representations belong together. Make sure that students can offer an explanation for each match, including for their last pair (other than because the description and the graph are the only pair left). See some possible explanations in the Monitoring Tip.
- Next, ask students to share the points that they think would be helpful for determining the domain and range of each function. If students gesture to the intercepts, a maximum, or a minimum on a graph but do not use those terms to refer to points, ask them to use mathematical terms to clarify what they mean.

> **RESPONSIVE STRATEGY**
>
> Use this routine to support whole-class discussion as students explain how they matched the function description to the graph. Display the following sentence frames for all to see: "____ matches the graph ____ because . . ." and "I noticed ____, so . . . ." Encourage students to challenge each other when they disagree. The student frames will help students consider the domain and range of functions as they connect written descriptions with graphical representations.
>
> Discussion Supports (MLR8)

Step 4

- Display the following information, needed for question 3, for all students to see.
  - The child is given 30 seconds on the swing.
  - While the child is on the swing, an adult pushes the swing a total of five times.
  - The swing is 1.5 feet (18 inches) above ground.
  - The chains that hold the seat and suspend it from the top beam are 7 feet long.
  - The highest point that the child swings up to is 4 feet above the ground.
- If time is limited, ask each partner to choose two functions (different from their partner's) and write the domain and range only for those functions.

> **RESPONSIVE STRATEGY**
>
> Use color coding and annotations to highlight connections between representations in a problem. For example, invite students to highlight the input in each description using one color, label the horizontal axis of each graph, and highlight the label in the same color. Then they can repeat the process for the outputs and the vertical axes.
>
> Supports accessibility for: Visual-spatial processing

Student Task Statement

3. Once you receive the information you need from your teacher, describe the domain and range that would be reasonable for each function in this situation.

Step 5

- Select students to share the domain and range for each function and share their reasoning. Record and display their responses for all to see.
- One key point to highlight is that the range of a function could be a single value (say 7, as shown in graph C), a bunch of isolated values (say, only some whole numbers, as shown in graph B), all values in an interval (say, all values from 1.5 to 4, as shown in graph D, or all values between 0 and 30, as in graph A), or a combination of these. The domain of a function may also be limited in similar ways.

DO THE MATH

PLANNING NOTES

Activity 2: Back to the Bouncing Ball (*10 minutes*)

Instructional Routine: Discussion Supports (MLR8)	
Building On: NC.M1.F-IF.2	**Addressing:** NC.M1.A-REI.10; NC.M1.F-IF.4; NC.M1.F-IF.5

In this activity, students continue to interpret a graph of a function in terms of a situation and relate the features of the graph to the domain and range of the function. The context is a familiar one, allowing students to focus their reasoning on domain and range.

Unlike the activity about a child on a swing, the graph includes a scale on each axis and the coordinate pairs of some points, allowing students to identify the range more definitively. However, the graph is a partial representation of the function, as it does not show what happens after a dropped ball hits the ground the fifth time. When describing the domain, students need to attend to what is reasonable in this situation (that is, noting that the ball likely does not just stop after the fifth bounce).

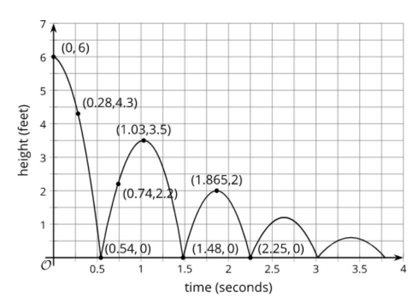
Step 1

- Have students remain in their same groups.
- Provide students 2–3 minutes of quiet work time and then time to share their thinking with their partner. Students should take turns with their partners when sharing their thinking.

Student Task Statement

A tennis ball was dropped from a certain height. It bounced several times, rolled along for a short period, and then stopped. Function H gives its height over time.

Here is a partial graph of H. Height is measured in feet. Time is measured in seconds.



Use the graph to help you answer the questions.

Be prepared to explain what each value or set of values means in this situation.

1. Find $H(0)$.
2. Find the value of x when $H(x) = 0$.
3. Describe the domain of the function.
4. Describe the range of the function.

Are You Ready For More?

In function H, the input was time in seconds and the output was height in feet.

Think about some other quantities that could be inputs or outputs in this situation.

1. Describe a function whose domain includes only integers. Be sure to specify the units.
2. Describe a function whose range includes only integers. Be sure to specify the units.
3. Sketch a graph of each function.

Step 2

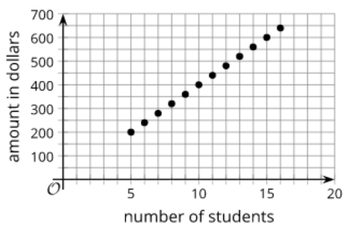
- Facilitate a whole-class discussion focused on how students reasoned about the domain and the range for the function. As students share explanations of their reasoning about the domain and the range for the function, use *Discussion Supports* to encourage students to press for details and precision in language in peers' responses by asking questions such as, "Can you explain this using points on the graph?" or "Are there values for the domain and range not shown on the graph? How do you know?"
- Highlight explanations that account for what is realistic in the context.
  - The domain tells us all the possible amounts of time that passed since the moment the tennis ball was dropped until it stopped rolling.
  - The range includes all the possible heights of the tennis ball from the time it was dropped until the time it stopped rolling.
- Ask students if there are points on the graph whose coordinates are particularly useful for identifying the domain and range of the function. (Possible responses/prompts: The heights of the bounces? The points where the ball hits the ground? Points between the two? (probably not) Others? (again, probably not, although students may point out that in other situations not involving a bouncing ball it will be important to look at the points with the least and greatest horizontal coordinates).)
- Emphasize that, just like in the activity about the swing, some points and features on a graph can give us more information than others about possible input-output values of a function.

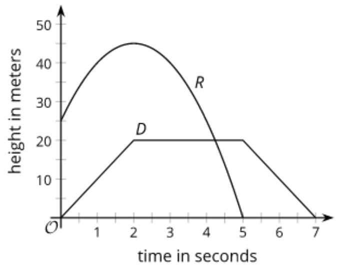
DO THE MATH	PLANNING NOTES

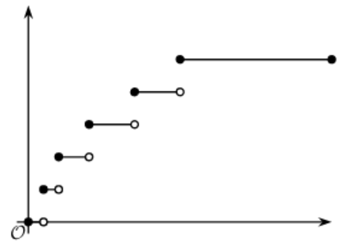
Lesson Debrief (5 *minutes*)

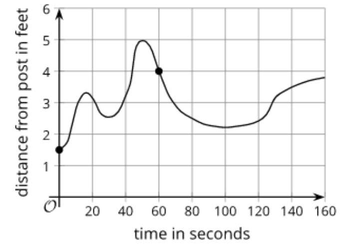
The purpose of this lesson is for students to use key features of graphs to gain information about the domain and range of a function.

Display several familiar graphs of functions from this unit. Here are some examples.









Choose whether students should first have an opportunity to reflect on the questions in their workbooks or talk through the prompts with a partner.

Ask students to examine the graphs and use them to help summarize what graphs can tell us about the domain and range of functions. Ask students to complete the following prompts as thoroughly as they can.

- I can learn about the domain and range of a function from a graph by looking for...
- A graph may not always show all that is needed to fully describe the domain and range, however. For example, it may not show...

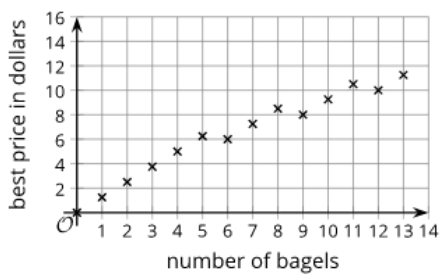
PLANNING NOTES

Student Lesson Summary and Glossary

The graph of a function can sometimes give us information about its domain and range.

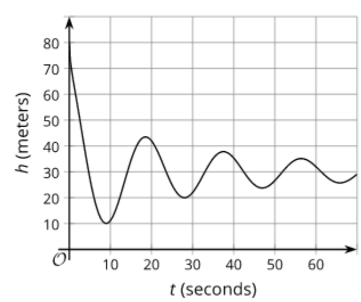
Here are graphs of two functions we saw earlier in the unit. The first graph represents the best price of bagels as a function of the number of bagels bought. The second graph represents the height of a bungee jumper as a function of seconds since the jump began.

What are the domain and range of each function?



The number of bagels cannot be negative but could include 0 (no bagels bought). The domain of the function therefore includes 0 and positive whole numbers, or $n \geq 0$.

The best price can be \$0 (for buying 0 bagels), certain multiples of 1.25, certain multiples of 6, and so on. The range includes 0 and certain positive values.



The domain of the height function would include any amount of time since the jump began, up until the jump is complete. From the graph, we can tell that this happened more than 70 seconds after the jump began, but we don't know exactly when.

The graph shows a maximum height of 80 meters and a minimum height of 10 meters. We can conclude that the range of this function includes all values that are at least 10 and at most 80.

Cool-down: A Pot of Water (*5 minutes*)

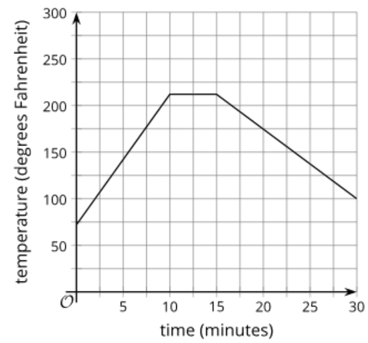
Addressing: NC.M1.A-REI.10; NC.M1.F-IF.4; NC.M1.F-IF.5

Cool-down Guidance: Press Pause
This is the last lesson in this unit. If students are still struggling with domain and range, examine errors in the cool-down and consider small-group instruction using the practice problems provided in this lesson.

Cool-down

The function W gives the temperature, in degrees Fahrenheit, of a pot of water on a stove t minutes after the stove is turned on. After 30 minutes, the pot is taken off the stove.

The graph of the function is shown.



1. Is 250 in the range of function W? Explain how you know.
2. Describe the range of the function.
3. Is there a valueof t that makes $W(t) = 0$ true? Explain how you know.

Student Reflection:

I think my math ability is:

a. Great and getting better
b. Okay, and still growing
c. A battle, but I am learning and growing

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

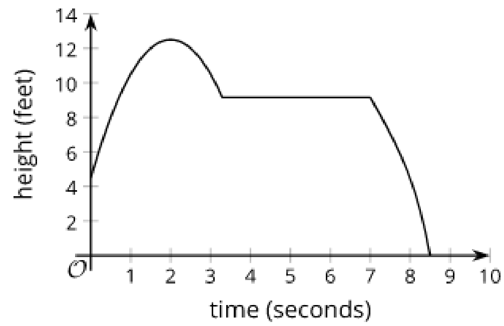
Consider the students' reflections on how they perceive their own math ability. How do you think you have influenced their thinking? What would you like to stop, start, or keep doing?

Practice Problems

1. A child tosses a baseball up into the air. On its way down, it gets caught in a tree for several seconds before falling back down to the ground.

   Select the **best** description of the range of this function.

   a. The range includes all numbers from 5 to 12.5.
   b. The range includes all integers between 0 and 12.5.
   c. The range includes all numbers from 0 to 8.5.
   d. The range includes all numbers from 0 to 12.5.



2. A newly planted cedar tree grows at a rate of about 3 inches per month, representing the height H after t months. The Carolina Panthers get 3 points for each field goal they make in a football game, representing their score in the game S after f field goals.

   a. What equations could represent $H(t)$ and $S(f)$?

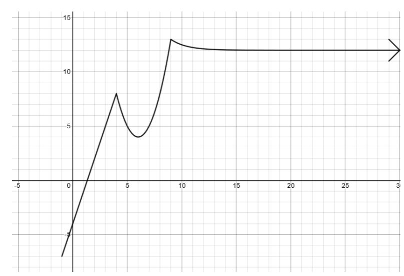
   b. What are possible domains for $H(t)$ and $S(f)$?

   c. What are possible ranges for $H(t)$ and $S(f)$?

   d. Explain any differences in the domains and ranges of the two functions.

3. Based on the graph of the function, select **all** the true statements.

   a. The domain and range of the function are all real numbers.
   b. Point $(4, 6)$ is a solution to the function.
   c. $y = -3$ is in the range of the function.
   d. $x = -3$ is in the domain of the function.
   e. Point $(9, 13)$ is a solution to the function.
   f. The range of the function is $-7 \le y \le 13$.



4. The graph H shows the height, in meters, of a rocket t seconds after it was launched.

   a. Find $H(0)$. What does this value represent?

   b. Describe the domain of this function.

   c. Describe the range of this function.

   d. Find $H(x) = 0$. What does this value represent?

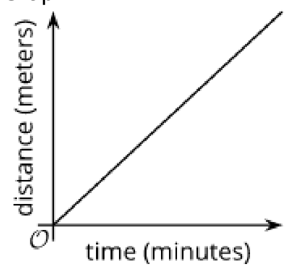
   (From Unit 5, Lesson 12)

5. Lin completes a 5K using a combination of walking and running. Here are four graphs that represent four possible situations. Each graph shows the distance, in meters, as a function of time, in minutes.

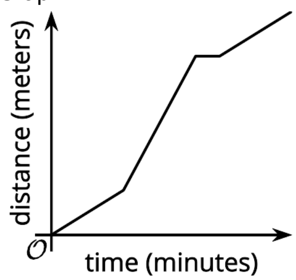
   Match each description with a graph that could represent it.

   a. Lin starts out running, but then slows down to a jog. After 10 minutes, she stops for a water break. She then runs the rest of the way.
   b. Lin starts the race walking, gradually getting faster and faster.
   c. Lin jogs at a steady pace for the entire race.
   d. Lin starts out walking, then moves to a steady run. After 15 minutes, she stops to stretch a cramped leg. Then, she walks the rest of the way.

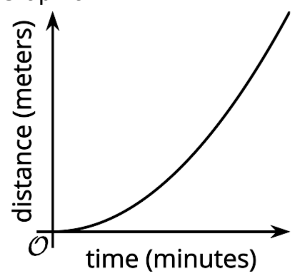
   Graph 1



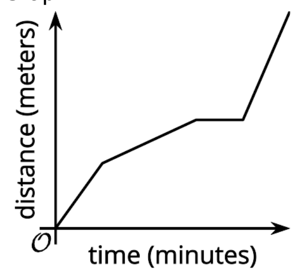
   Graph 2



   Graph 3



   Graph 4



   (From Unit 5, Lesson 7)

6. Mai has to decide between two cafeteria meal plans. Under plan A, each meal costs \$2.50. Under plan B, one month of meals costs \$30.

   a. Write an equation for function A, which gives the cost, in dollars, of buying n meals under plan A.

   b. Write an equation for function B, which gives the cost, in dollars, of buying n meals under plan B.

   c. Mai estimates that she'll buy 15 meals per month. Which meal plan should she choose? Explain your reasoning.

   (From Unit 5, Lesson 5)

7. C gives the cost, in dollars, of a cafeteria meal plan as a function of the number of meals purchased, n. The function is represented by the equation $C(n) = 4 + 3n$.

   a. Find a value of n for which $C(n) = 31$.

   b. What does that value of n tell you about the cafeteria meal plan?

   (From Unit 5, Lesson 5)

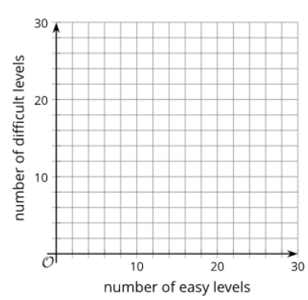
8. Kiran is playing a video game. He earns three stars for each easy level he completes and five stars for each difficult level he completes. He completes more than 20 levels total and earns 80 or more stars.

   a. Create a system of inequalities that describes the constraints in this situation. Be sure to specify what each variable represents.

   b. Graph the inequalities and show the solution region.

   c. Then, identify a point that represents a combination of stars and levels that is a solution to the system.

   d. Interpret the point $(5, 6)$ in the context of this situation and determine how many stars Kiran earns based on this point.



   (From Unit 3)

Lesson 14: Post-Test Activities

PREPARATION

Lesson Goal	Learning Targets
• Provide students the opportunity to reflect and share feedback on their own progress and on the culture and instruction happening in the class.	• I can reflect on my progress in mathematics. • I can share feedback that can help make me and my teacher grow.

Lesson Narrative

This lesson, which should occur after the Unit 5 End-of-Unit Assessment, allows for students to reflect on the unit, share feedback, conference with the teacher, and engage in activities that support the work of the upcoming unit.

Gathering student feedback is a powerful and strategic way to learn about students and improve instructional practices. It also creates student and family buy-in and centers students as decision makers and problem solvers in their own learning.

What do you hope to learn about your students during this lesson?

Agenda, Materials, and Preparation

- **Activity 1** (*20 minutes*)
  - End-of-Unit 5 Student Survey (print 1 copy per student)
- **Activity 2** (*25 minutes*)
  - Who are They? pictures (print 1 set of pictures per small group)
  - Who are They? answer key (print 1 set per small group)

LESSON

Activity 1: End-of-Unit 5 Student Survey (*20 minutes*)

The End-of-Unit 5 Student Survey is a critical opportunity for teachers to gather low-stakes, non-evaluative feedback to support equity and instructional pedagogy. The survey is also highly beneficial for students as it is designed to encourage self-awareness, self-management, social awareness, relationship skills, and responsible decision making. Provide students a chance to quietly and independently complete this survey after they complete their testing.

One-on-One Conferences

Conducting one-on-one conferences with students, using the surveys as a data point, is encouraged. These conferences can be done as students complete their surveys and are engaging in Activity 2. Potential conference topics include:

- student responses to the daily student reflections
- student response to the end-of-unit student survey (as students finish them)
- executive functioning skills
- student learning contracts
- goal setting and self-evaluation

Activity 2: Who Are They? (*25 minutes*)

This is an activity meant to help develop mathematical identity while also addressing common biases held in mathematics.

Step 1

- Have students arrange themselves in small groups or use visibly random grouping.
- Give each small group the printout of Who Are They? pictures. Have students spend about 5 minutes viewing each picture and making a prediction on what they believe is the profession of the person in the picture. Encourage students to compare their predictions with each other via small groups.

Step 2

- Have each group pick up one copy of the Who Are They? answer key to reveal the professions for each picture.

Step 3

- Encourage students to engage in a small-group discussion using the questions below.

Consider having students record some key discussion points as an artifact.

Student Task Statement

1. Review the “Who Are They?” pictures and predict each person’s profession.

Daniel Akyeampong:	Charlotte Baidoo:	Marjorie Lee Brown:	Carlos Castillo-Chavez:
Lorin Crawford:	Tiffany Kelly:	Autumn Kent:	Susan Murphy:

2. Read over the answer key to reveal each person’s profession.
3. Engage in a small-group discussion using the following prompts:
    - What made you predict the way you did?
    - Is there a specific way you think of mathematicians?
    - Who was most surprising to you and why?
    - Were any predictions correct?
    - What are the implications of making an assumption that mathematicians do not look like us?

TEACHER REFLECTION

As you finish up this unit, reflect on the norms and activities that have supported each student in learning math. List ways you have seen each student grow as a young mathematician throughout this work.

List ways you have seen yourself grow as a teacher over the last few months.

What will you continue to do and what will you improve upon in Unit 6?